# Adverse Selection, Slow Moving Capital and Misallocation*

**William Fuchs**
UC Berkeley

**Brett Green**
UC Berkeley

**Dimitris Papanikolaou**
Northwestern University and NBER

July 16, 2014

**Abstract**

We embed an adverse selection friction into a dynamic, general equilibrium model with heterogeneous capital and sectoral productivity shocks and study its implications for aggregate dynamics. A key insight is that in a dynamic economy, adverse selection leads to delays in capital reallocation and thus slow recoveries from shocks, even those that do not affect the economy's potential output. The information friction provides a micro-foundation for convex adjustment costs, and our model links the magnitude of these costs to the underlying economic environment. The model predicts that the (endogenous) costs to reallocation increase with dispersion in productivity and decrease with the interest rate, the frequency of sectoral shocks and households consumption smoothing motives. When households are risk averse, misallocation serves as a hedge against future shocks and can lead to persistent misallocation.

*We are grateful to Andy Atkeson, Marty Eichenbaum, Andrea Eisfeldt, Joao Gomes, Zhiguo He, Pierre-Olivier Gourinchas, Boyan Jovanovic, Arvind Krishnamurthy, Lars-Alexander Kuehn, Pablo Kurlat, Adriano Rampini and Andrzej Skrzypacz for useful comments and feedback. We also thank conference participants at the Duke-UNC Asset Pricing Conference, NASM, NBER, FARFE, Texas Finance Festival, SED, and SITE. Finally, we thank seminar participants at Berkeley, University of Chicago, Duke University, EPFL, Northwestern University, San Francisco Federal Reserve, Toulouse School of Economics, UBC, and UCLA for useful conversations and comments. We are grateful to Andreas Neuhierl for excellent research assistance.

An important factor in determining aggregate productivity in an economy is the allocative efficiency of its resources. For example, there is growing consensus among economists that misallocation is large enough to explain a significant part of the TFP gap across rich and poor countries.[1] Part of this persistence in misallocation has been linked to the failure of markets.[2] In this paper, we explore an adverse-selection based theory of misallocation due to a failure in the market for used capital. We show that the information friction leads to an equilibrium with slow movements in capital flows with dynamics resembling those arising in models with convex adjustment costs.

Our starting point is that reallocating productive resources requires a transaction between two parties. We refer to these resources as 'capital', which may represent physical capital, human capital, or existing matches between physical and human capital – such as a division of a firm – whose productivity cannot be verified or contracted upon. Our model consists of a two-sector dynamic economy in which sectoral productivity shocks arrive randomly, creating a reason for reallocating capital from the less productive sector to the more productive one. Capital reallocation takes place in a competitive market; the 'sellers' are firms in the less productive sector who own capital and the 'buyers' are firms in the more productive sector who demand capital. In the absence of any frictions, capital is immediately reallocated to the more productive sector following a productivity shock.

We introduce an information asymmetry by allowing capital to vary in 'quality' and firms to privately observe the quality of the capital they own and operate. The output of a unit of capital depends on its own quality as well as the productivity of the sector to which it is allocated. Firms looking to purchase capital are therefore at an information disadvantage. In a static environment, this friction can lead to a complete breakdown in the market for capital (Akerlof, 1970). Within our dynamic economy, the adverse selection problem translates into a slow moving reallocation process; capital moves gradually from the less productive sector to the more productive one. Following a productivity shock, firms who own lower quality capital are eager to sell, while firms with higher quality capital are content to continue operating their capital in the less productive sector. Firms in the more productive sector (i.e., buyers) recognize this and as a result, lower quality capital is reallocated more quickly, but at a lower price. Delays in reallocation generate real economic costs because capital – especially higher quality capital – continues to operate in the less productive sector following a productivity shock.

To isolate the implications of our mechanism, we begin by assuming households are risk

[1]For instance, Hsieh and Klenow (2009) compare the dispersion in productivity between the US, India and China and find substantially lower dispersion in the US. Using a fairly general model, they argue that if the dispersion in TFP in India and China were equal to US levels, TFP would be 30-60% higher.

[2]See, for example, Banerjee and Duflo (2005).

neutral; hence the interest rate is constant and equal to the subjective discount rate. We examine the process of capital reallocation from the less productive to the more productive sector following a *permanent* productivity shock and derive a link between the production technology and the rate at which capital reallocation takes place. With this link, the model yields qualitative predictions about the reallocation dynamics. For example, the rate of reallocation increases over time when the factor inputs (i.e., quality and productivity) are complementary, and decreases over time when inputs are substitutes. We relate these findings to commonly used convex adjustment cost specifications. Depending on the degree of complementarity between quality and productivity, the resulting equilibrium dynamics are similar to convex adjustment costs that penalize changes in either the level or the growth rate of capital.[3]

Perhaps surprisingly, when shocks are *transitory*, capital reallocation may be more efficient. A novel mechanism in our setting is that costly reallocation can serve to mitigate adverse selection. The intuition is that a firm looking to purchase capital today internalizes the inefficiency associated with selling capital in the future. As a result, firms care not only about the quality of their capital, but also on its (endogenous) liquidity. This leads to an *illiquidity discount* in capital prices, which in turn influences a firms decision of when to sell its capital. In equilibrium, the discount and the rate of reallocation are jointly determined. Higher quality capital takes longer to be reallocated and is therefore associated with a larger discount. The higher discount lowers the benefit of mimicking higher quality types, thus inducing lower quality types to trade more quickly.

We introduce households with CRRA utility to explore the general equilibrium implications and illustrate how our results extend to this case in which the interest rate is endogenously determined. In addition, we highlight several new insights resulting from equilibrium effects. First, the desire to smooth consumption increases the firms' cost of delay and translates into faster reallocation. Second, the model predicts that large downturns are followed by fast recoveries whereas smaller negative shocks are followed by slower recoveries. Both of these predictions are in contrast to the predictions of models with convex adjustment costs. Third, with transitory shocks and risk averse households, firms have a diversification motive to remain in the inefficient sector. As a result, the rate of reallocation may reach zero prior to all capital being reallocated leading to long-run persistence in misallocation.

In sum, our model delivers persistence in misallocation in response to a sectoral shock, and

[3] For example, adjustment cost specifications that penalize change in the fraction of capital stock – implying a declining rate of reallocation – are consistent with the model with adverse selection in which the economic gain from reallocation is decreasing in capital quality. Specifications that penalize the change in the rate of reallocation – implying an increasing rate of reallocation – are consistent with a model in which the economic gain from reallocation is increasing in capital quality.

consequently a drop in aggregate total factor productivity and output. The dynamics implied by our model are qualitatively similar to those that would obtain in a model with convex adjustment costs. However, in our setting the costs of reallocating capital are endogenous to the economic environment; hence our model can help interpret recent empirical work that argues for the importance of time-variation in reallocation costs. For instance, Eisfeldt and Rampini (2006) argue that counter-cyclical reallocation costs are needed to reconcile the fact that reallocation activity is pro-cyclical, while the gains from reallocation – measured as the dispersion in productivity or Tobin's $Q$ is counter-cyclical.

We illustrate how parameter shifts in our model can lead to time-variation in the costs of reallocating capital. We conduct impulse responses to shifts in the dispersion in capital quality and the level of the interest rate. When the dispersion of capital quality increases, the degree of adverse selection increases which reduces the allocative efficiency and therefore aggregate productivity and output. Perhaps surprisingly, a similar result obtains in response to reduction in the interest rate. A reduction in the interest rate reduces the cost that the low types face in emulating the high types behavior, thus worsening the adverse selection problem and therefore increasing the costs of reallocation.

Most of the paper focuses on the reallocation of existing capital, which accounts for a significant fraction of new firms investments. Eisfeldt and Rampini (2007) report that the fraction of capital expenditures that are comprised of used capital varies from 10% to 30% across firm size deciles. However, the economic forces can apply more broadly. In Section 5, we show that our mechanism can also lead to frictions in the creation of new projects and offer an empirical design to test this theory. We introduce entrepreneurs who have the ability to create new units of capital – projects or firms – upon the arrival of an investment opportunity. Entrepreneurs are heterogenous in ability: highly skilled entrepreneurs create projects of higher quality. Entrepreneurs have limited capacity or financial capital, so in order to start a new project they must first sell their existing one, about which they have private information. Consistent with the model of reallocation, two features of the equilibrium arise. First, entrepreneurs with more profitable existing projects wait longer before selling them. Second, the market correctly interprets the length of delay as a signal of quality and prices adjust accordingly. This model also generates delayed response to new investment opportunities and gradual increases in the measured productivity of the new sector. When entrepreneurs ability is sufficiently persistent across investment opportunities, aggregate measured productivity drops in response to innovations. This obtains because the first adopters of the new technology are the lower-ability entrepreneurs.

Since the model's predictions pertain to unobservable characteristics, gathering data that facilitates a direct test of the mechanism is inherently challenging.[4] In order to do so, we focus on the change in ownership from entrepreneurs to investors following a firm's initial public offering (IPO). We construct an empirical test of our mechanism by relating the length of time elapsed between a firm's incorporation and its initial public offering (IPO) to post-IPO measures of it's profitability, controlling for observable characteristics at the time of the IPO. We interpret these post-IPO changes in profitability as being correlated with the quality of the firm. We find that the age of the firm at the time of the IPO is strongly related to post-IPO measures of profitability, consistent with the idea that the owners of high-quality firms wait longer before selling. We find no corresponding relation between firm age at IPO and subsequent changes in firm valuations, suggesting that these post-IPO increases in profitability are not news to investors, consistent with the idea that the price is revealing at the time of the sale.

**Related Literature**

Our paper contributes to a small but growing literature that introduces adverse selection into dynamic macro-finance models. The most closely related papers are Eisfeldt (2004), House and Leahy (2004) and Kurlat (2013). Eisfeldt (2004) and House and Leahy (2004) study the problem of equity issuance and consumer's choice of a durable good in an environment with adverse selection. Both papers find that increasing the variance of the underlying shock increases non-informational motives for trade and thus ameliorates the adverse selection problem. Kurlat (2013) studies a setting in which entrepreneurs have private information about their projects. He shows that this is mathematically equivalent to a tax on capital, which leads to an amplification mechanism in response to aggregate shocks.[5] In all of these papers, the informational asymmetry last for one period. By contrast, the duration of the information asymmetry is endogenously determined in our model and our focus is on how the information friction itself generates persistence in aggregate dynamics. Our work is also complementary to Guerrieri and Shimer (2014) and Chang (2012) who consider economies in which many markets exist simultaneously and firms with higher quality assets trade at higher prices but with lower probability. Our approach differs from theirs in several respects. First, we analyze a production economy with a single marketplace where capital is traded rather than an endowment economy with segmented markets for assets. Second, in our model,

[4] We should emphasize that the reallocation decision of firms in our model operates based on *unobservable* characteristics. Absent this distinction, some of the model's predictions may appear to run counter to what intuition would suggest. Specifically, one may naturally expect that, in contrast to our model, 'higher types' should reallocate faster than 'lower types'. However, this intuition refers to *observable* characteristics, in which case higher types can receive a higher price regardless of the timing of their reallocation decision, then they will naturally reallocate more quickly than lower types.

[5] See also Bigio (2013), who incorporates a labor market in a related setting.

households and firms anticipate the potential for aggregate shocks and our focus is on the dynamics of aggregate quantities in response to these shocks; the distribution of capital across sectors changes over time, thereby affecting total output and productivity. On the more technical side, our model takes place in continuous-time and with a continuum of types; this allows us to derive equilibrium objects through ordinary differential equations, which allows us to compute examples in closed form and facilitates analytic tractability.

More broadly, our work is related to the voluminous literature studying the effect of information asymmetries on the market for capital. In financial economics, models with adverse selection are commonly used to study the sale of claims on firms' capital (see, for instance Leland and Pyle, 1977; Myers and Majluf, 1984; Brennan and Kraus, 1987; Lucas and McDonald, 1990; Korajczyk, Lucas and McDonald, 1991). Several papers papers provide empirical evidence documenting a significant role for adverse selection in a variety of economic environments. Specifically, Gibbons and Katz (1991) focus on the labor market, Lizzeri and Hendel (1999) study the market for durable goods, Finkelstein and Poterba (2004) examine the market for insurance, and Michaely and Shaw (1994) focus on financial markets.

The fact that adverse selection can generate delays in trade between buyers and sellers is well understood within the dynamic adverse selection literature. Janssen and Roy (2002) derive a competitive equilibrium in which the price mechanism sorts sellers of different qualities into different (discrete) time periods. Hörner and Vieille (2009); Fuchs, Oëry and Skrzypacz (2014) investigate the implications of public versus private offers in a discrete-time model. Daley and Green (2012, 2014) study trade dynamics in setting where information about the seller's quality is revealed gradually. Our baseline model builds on Fuchs and Skrzypacz (2013), who study the costs and benefits of temporarily closing the market. Our contribution to this literature is to embed dynamic adverse selection into a production economy and to study the equilibrium quantity dynamics in a general equilibrium environment.

Clearly, the theory we propose here is not the only one that can generate disruptions in the efficient allocation of resources. The existing literature is rich with alternative theories; physical (convex) costs, search, financial frictions, learning, time-to-build and other factors are likely to be important components in the allocation of new and existing capital (see Banerjee and Duflo, 2005, for a review of the literature on causes of misallocation). Indeed, one benefit of specifying an exogenous cost function is that it can potentially embed all these considerations. By contrast, by focusing on a particular friction, we are able to examine how these costs vary endogenously with the economic environment.

The remainder of the paper is organized as follows. In Section 1, we illustrate how adverse selection generates slow movements in capital across sectors and describe the relation to various convex adjustment cost models. In Section 2, we embed the mechanism into a stationary, general equilibrium model. Section 3 analyzes equilibrium of the model with risk-neutral households, studies the aggregate dynamics in response to transitory shocks as well as impulse responses to structural shifts. Section 4 extends our results to a setting with risk-averse households, which provides several new insights. Section 5 extends the model to study new investment. Section 6 presents an empirical test and evidence that is consistent with our theory. Section 7 concludes. Proofs are located in the Appendix A.

# 1 A Motivating Example

To illustrate the main ideas in the paper, we start with an example. Consider an economy with two productive sectors, $i \in \{A, B\}$. Households are risk neutral and have an infinite horizon; hence the interest rate is fixed at $r$. There is a mass $M > 1$ firms in each sector. Firm cannot migrate across sectors. Firms maximize total discounted profits, which includes the purchase or sale of any capital.

There is a unit mass of capital. Capital is heterogenous in its productivity, also referred to as *quality* or *type* and denoted by $\theta$, which is distributed according to a uniform distribution with support $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_{++}$. Output of the capital stock depends on sector productivity $z_i$ and capital quality. Quality is observable only to the firm who owns and operates the capital. If the firm does not have any capital, it remains idle and produces zero output. For simplicity, we assume here that capital does not depreciate and there is no inflow of investment (the model in Section 2 incorporates such features).

A unit of capital of quality $\theta$, henceforth a "$\theta$-unit," operated by a firm in sector $i$, generates a flow of output per unit time equal to

$$\pi_i(\theta) = (\beta\theta^\alpha + (1-\beta)z_i^\alpha)^{\frac{1}{\alpha}}, \tag{1}$$

where $\beta$ captures the importance of capital quality in production, and $(1-\alpha)^{-1}$ represents the elasticity of substitution between capital quality and sector productivity.

We are interested in the process by which capital is reallocated from sector $A$ to sector $B$. Therefore, assume that at $t = 0$, all capital is allocated to firms in sector $A$ and that sector productivity is higher in sector $B$, $z_B > z_A$, perhaps due to a demand shock or recent technological innovation in sector $B$. Prior to analyzing the role of adverse selection, we first establish the frictionless benchmark. We then illustrate the key aspects of our mechanism and how adverse selection can endogenously generate reallocation costs. Finally, we compare our predictions to a model with exogenously specified costs to reallocating capital.

## 1.1 Benchmark: Frictionless Environment

In the absence of any reallocation costs, a social planner will immediately reallocate all capital from sector $A$ to sector $B$. In a decentralized economy, the same outcome obtains without the information friction. To see this, suppose that $\theta$ is perfectly observable and therefore prices can be conditioned on capital quality $\theta$.

At any point in time, a sector $B$ firm is willing to pay up to $\pi_B(\theta)/r$ to buy a $\theta$-unit of capital. Since capital is scarce and sector $B$ firms are identical and competitive, the price for a $\theta$-unit will get bid up to exactly this amount. Each sector $A$ firm will sell at $t = 0$ at a price equal to the present value of the output the capital generates in sector $B$. Since there is no informational friction and there are gains from reallocation, all capital is immediately and efficiently reallocated.

## 1.2 Heterogeneous Capital and Adverse Selection

Next, we introduce the informational friction: capital is heterogenous in its quality ($\underline{\theta} < \overline{\theta}$), which is privately observed by the firm who owns it. We study the competitive equilibrium of the decentralized economy in which reallocation decisions are made by firms. In order for a unit of capital to be reallocated, a transaction must take place: a firm in sector $B$ must purchase the capital from a firm in sector $A$. This occurs in a dynamic marketplace; at every $t \geq 0$ a firm in sector $A$ who wishes to sell its unit of capital can trade with firms in sector $B$ who wish to purchase capital. There are no institutional frictions in the market (e.g., transactions costs or search). The only friction is an informational one. That is, buyers cannot observe the quality of capital in the market prior to purchasing it (or, alternatively, it is too costly to do so). Therefore, sector $B$ firms face a potential adverse selection problem in the market for capital. We restrict attention to the case where the adverse selection friction is binding, $\pi_A(\overline{\theta}) > \int \pi_B(\theta) dF(\theta)$, that is, the case in which a firm with the highest quality capital in sector $A$ would prefer to retain its capital rather than trade at the average value to firms in sector $B$.

A competitive equilibrium of this environment can be characterized by (1) a path of prices $P_t$, and (2) the time at which each unit of capital is reallocated, denoted by $\tau(\theta)$.[6] We formalize our notion of equilibrium in Section 2 (see Definition 1). Roughly, it requires that (i) given the path of prices, sector $A$ firms with capital choose the optimal time to trade, (ii) firms in sector $B$ make zero expected profits and (iii) that the market for capital clears.

Since quality is unobservable, prices cannot be conditioned on $\theta$ and the first-best reallocation cannot be part of an equilibrium. To see why, suppose that all sector $A$ firms sell

[6] We allow for the possibility that certain types of capital are never reallocated, in which case $\tau(\theta) = \infty$.

their capital at $t = 0$. For sector $B$ firms to break even requires that $P_0 = \frac{1}{r} \int \pi_B(\theta) dF(\theta)$. But given this price, a firm with capital of quality $\overline{\theta}$ in sector $A$ would prefer to retain her capital. An alternative conjecture is that all firms with capital quality below some threshold trade at $t = 0$. In this case, the remaining capital in sector A is of discretely higher quality and the equilibrium price would jump upward. Clearly then firms that sold capital at $t = 0$ did so suboptimally. Hence, we have ruled out a mass of reallocation at date zero.

Next, we construct an equilibrium in which the time at which capital is reallocated reveals its quality; firms trade off the immediate gains from reallocation versus preserving the option to sell it in the future. Firms with lower quality capital are effectively more anxious to sell – since their capital is less productive – and do so sooner than firms with high quality capital. Since higher quality capital gets reallocated later, the market price of capital will gradually increase over time.

To construct this equilibrium, let $\chi_t$ denote the quality of capital that is reallocated at date $t$. In order for sector $B$ firms to break even, it must be that

$$P_t = \frac{\pi_B(\chi_t)}{r}. \tag{2}$$

For this to be an optimal strategy, the firm who owns a $\chi_t$-unit of capital must be locally indifferent between trading immediately or waiting an instant for a higher price:

$$\underbrace{rP_t - \pi_A(\chi_t)}_{\text{Cost of Delay}} = \overbrace{\frac{d}{dt} P(t)}^{\text{Benefit}}. \tag{3}$$

The left hand side of (3) corresponds to the cost that a firm with a $\chi_t$-unit in sector $A$ gives up by delaying trade. Using (2), the right hand side can be rewritten as:

$$\frac{d}{dt} P_t = \frac{\pi_B'(\chi_t)}{r} \dot{\chi}_t, \tag{4}$$

where $\dot{\chi}_t = \frac{d\chi_t}{dt}$ represents the rate of skimming. Since the distribution over types is uniform, $\dot{\chi}_t$ is proportional to the rate at which capital is reallocated to the more productive sector. Combining (3) and (4), we have that

$$\dot{\chi}_t = \frac{r\left(\pi_B(\chi_t) - \pi_A(\chi_t)\right)}{\pi_B'(\chi_t)} \tag{5}$$

This differential equation characterizes the equilibrium rate at which capital transitions to sector $B$. It is based on the first two equilibrium requirements, (i) that sector $A$ firms optimize their selling decisions and (ii) that sector $B$ firms break even. One immediate observation from (5) is that the rate of reallocation is proportional to the gains from doing so (the productivity differential $\pi_B - \pi_A$), relative to a firms' benefit of delaying in order to get a higher price (the sensitivity of price to capital quality $\pi'_B$). Another observation is that the rate of reallocation is proportional to the interest rate; the larger is $r$, the more costly it is for firms to delay reallocation and the hence the faster it occurs.

The boundary condition is pinned down by the market clearing condition, which requires the price at time zero to be at least $\pi_B(\underline{\theta})/r$. This implies that the lowest quality capital trades immediately

$$\chi_0 = \underline{\theta}. \tag{6}$$

For any set of production technologies $\{\pi_A, \pi_B\}$, equations (5) and (6) pin down the equilibrium reallocation dynamics.

In sum, adverse selection inhibits the reallocation of capital, resulting in a slow transition of resources to the more productive sector. The equilibrium dynamics depend, in part, on the production technology and specifically, on the elasticity of substitution between capital quality and productivity. Using our CES formulation, we focus on three values for this elasticity, $\alpha \in \{0, 1, 2\}$.

First, as $\alpha \to 0$, the production technology tends to a Cobb-Douglas. In this case, the gains from reallocation are increasing with quality. Equation (5) becomes

$$\dot{\chi}_t = \kappa \chi_t,$$

where $\kappa = \left(1 - \left(\frac{z_A}{z_B}\right)^{\beta}\right)(1-\beta)^{-1} r$. Combining with (6), the solution is given by

$$\chi_t = \underline{\theta} e^{\kappa t},$$

where the above holds for $t \leq \tau(\overline{\theta})$, where $\tau \equiv \chi^{-1}$. Hence, the equilibrium reallocation rate is *increasing over time* until $\tau(\overline{\theta})$, at which point, all capital has been reallocated to sector $B$ and the transition dynamics terminate.

Second, in the case $\alpha = 1$, the production technology is linear, and hence, there are constant gains from reallocation. For $t < \tau(\overline{\theta})$, equation (5) becomes

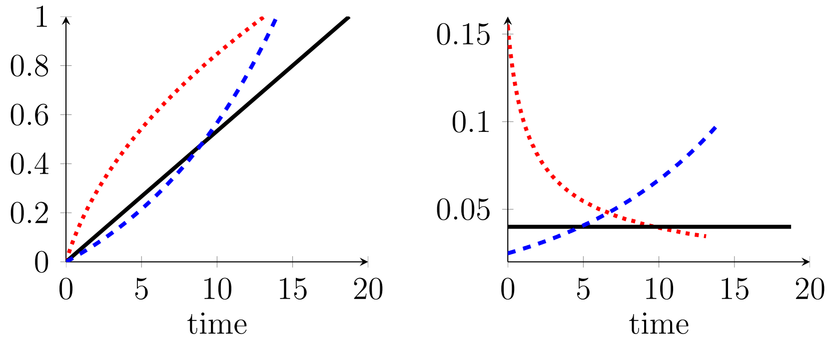
$$\dot{\chi}_t = \left(\frac{1-\beta}{\beta}\right)(z_B - z_A) r.$$

Since the right hand side is a constant, the equilibrium reallocation rate is *constant over time*.

Combining with (6), the solution is given by

$$\chi_t = \underline{\theta} + \left(\frac{1-\beta}{\beta}\right)(z_B - z_A)rt.$$

For the case that $\alpha = 2$, the differential equation does not admit an analytic solution, however, it is straightforward to compute it numerically. Moreover, it is easy to show that the equilibrium rate in this case will be *decreasing over time* (see Proposition 1.1).




(a) Fraction of capital reallocated (b) Rate of reallocation

**Figure 1:** Equilibrium reallocation with CES production technology for $\alpha = 0$ (blue dashed line), $\alpha = 1$ (black solid line) and $\alpha = 2$ (red dotted line). The left panel illustrates the capital quality that switches at time $t$, the right panel illustrates the rate at which capital is reallocated.

We plot the implied reallocation dynamics for the three cases in Figure 1. As we see in panel (a), the quality of capital that is reallocated increases over time in all three cases. This property is true regardless of the production technology; lower quality capital will reallocate sooner than higher quality capital for all specifications of the model. More importantly, panel (b) shows that the qualitative features of the equilibrium reallocation rate depend on the elasticity of substitution between factors. In terms of capital stock, the case with constant gains form trade ($\alpha = 1$) implies a constant rate of reallocation and linear change in capital stock. By contrast, the case of decreasing gains from trade ($\alpha = 2$) generates strictly concave dynamics for the capital stock, whereas the case with increasing gains from trade ($\alpha = 0$) the model generates a convex path for the capital stock.

The next proposition formalizes the findings illustrated in Figure 1:

**Proposition 1.1.** *Until all capital has been reallocated to the efficient sector:*

- *If $\alpha < 1$, the equilibrium rate of reallocation is strictly increasing over time.*
- *If $\alpha = 1$, the equilibrium rate of reallocation is constant over time.*

- *If $\alpha > 1$, the equilibrium rate of reallocation is strictly decreasing over time.*

### 1.3 Comparison to Exogenous Adjustment Cost specifications

To compare the predictions of our models to those of models with adjustment costs, we consider the case in which capital is homogeneous (i.e., $\underline{\theta} = \overline{\theta}$), but there are exogenous costs to reallocating capital. We examine three formulations for these costs, motivated by the adjustment costs specifications commonly used in the literature. We specify these costs as a function of the aggregate mass of capital being reallocated at a point in time and focus on the central planner's problem. We denote by $k$ the capital stock in sector $B$. The reallocation dynamics for these three cases are plotted in Figure 2.

The first formulation corresponds to the case where adjustment costs are convex in the rate of reallocation $\dot{k}$.

$$c(\dot{k}) = \frac{1}{2}\, c\, \left(\dot{k}\right)^2. \tag{7}$$

These costs are in line with the adjustment cost formulation in Abel (1983). We refer to this as the 'kdot' model. The second formulation is closely related to (7), except that it specifies the adjustment cost in terms of the growth rate of capital being reallocated,

$$c(k, \dot{k}) = \frac{1}{2}\, c\, \left(\frac{\dot{k}}{1-k}\right)^2 (1-k). \tag{8}$$

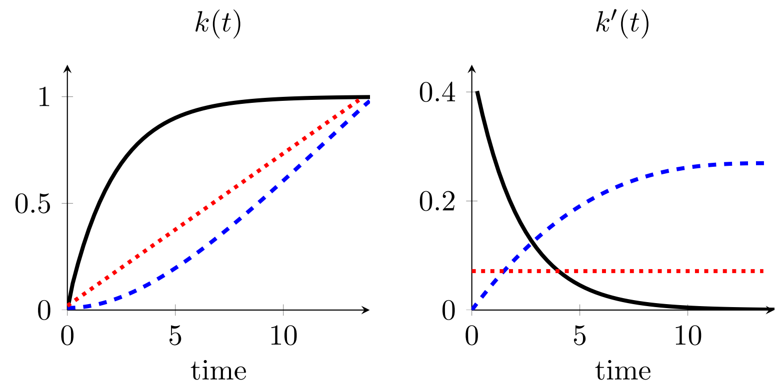
This type of adjustment costs is commonly used in the literature studying investment and reallocation dynamics (Abel and Eberly, 1994; Eisfeldt and Rampini, 2006; Eberly and Wang, 2009). We refer to these costs as the 'ik' model.

The last adjustment cost formulation penalizes *changes* in the flow rate of reallocation $\dot{k}$

$$c(\ddot{k}) = \frac{1}{2}\, c\, \left(\ddot{k}\right)^2, \tag{9}$$

and is based on the adjustment costs proposed by Christiano, Eichenbaum and Evans (2005), which penalize changes in investment. We refer to these costs as the 'idot' model.

We contrast our model's equilibrium dynamics to those implied by the models with different types of reallocation costs. As we compare Figure 1 to Figure 2, a striking similarity emerges. Specifically, that 'idot' models of adjustment costs generate an increasing rate of reallocation in line with the case with increasing gains from trade ($\alpha = 0$) , while 'ik' models of adjustment costs generate a decreasing rate of reallocation in line with the case with decreasing gains from trade ($\alpha = 2$). When the gains from reallocation are constant ($\alpha = 0$), the dynamics match those of the 'kdot' model. Relative to the first two formulations, the




(a) Fraction of capital reallocated

(b) Rate of reallocation

**Figure 2:** Comparison across the 'kdot' (red dotted), 'ik' (black solid), and 'idot' (blue dashed) adjustment cost models. The left panel illustrates the capital quality that switches at time $t$, the right panel illustrates the rate at which capital is reallocated. See Appendix for details.

'idot' model generates an S-shaped path for the capital stock and more delayed responses of capital flow to a sectoral productivity shock. The rate of capital reallocation in the 'ik' model spikes on impact and decays smoothly over time. By contrast, in the 'idot' model, the rate of capital reallocation increases slowly over time. This slow increase occurs because the formulation in (9) severely penalizes large adjustments to the rate. Christiano, Eichenbaum and Evans (2005) argue that this feature is crucial in explaining the response of aggregate investment to shocks.

In sum, we see that the equilibrium dynamics implied by each of these exogenous adjustment cost models are similar to those predicted by our model. Our framework can thus be interpreted as providing a micro-foundation for a variety of adjustment cost speculations. In contrast to models with exogenous costs, the cost of reallocation in our model is endogenous to the economic environment. In what follows, we generalize the model we outlined in Section 1.2 to allow for general production functions, transitory productivity shocks, and risk averse households.

# 2 Stationary Model of Capital Reallocation

Our motivating example in the previous section considers a single transitionary period since reallocation occurs only once. Here, we allow sectoral productivity to vary stochastically over time. In this case, firms will internalize the possibility of costly future reallocation in their decisions. Further, the frequency of these shocks affects the equilibrium prices of capital and, in turn, the reallocation dynamics.

*Technology.* Consumption goods are produced using capital. Capital can be located in one of two sectors (A and B). Capital is heterogenous in its quality, where quality is indexed by $\theta$. Quality is observable only to the owner of the capital unit; capital quality is distributed according to $F(\theta)$, which is continuous with strictly positive density over the support $\Theta = [\underline{\theta}, \overline{\theta}]$. The flow output of a unit of capital depends on its quality $\theta$, the sector in which it is currently allocated $i$ and the aggregate state $x$ according to

$$y_t^i(\theta) = \pi_i(\theta, x)dt,$$

where $\pi_i$ is strictly positive, increasing and twice differentiable in $\theta$, with uniformly bounded first and second derivatives.[7] We incorporate shocks to the model by allowing the production technology to vary stochastically over time. Specifically, we introduce a Markov switching process $X(\omega) = \{X_t(\omega), 0 \leq t \leq \infty\}$ defined on the underlying probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $X_t(\omega) \in \{x_A, x_B\}$ represents the state of the economy at date $t$. Henceforth, we omit the argument $\omega$, and use a $t$ subscript as a place holder for the argument $(t, \omega)$. Existing capital depreciates at rate $\delta$.

*Markets, Information and Prices.* Reallocation of capital occurs in a competitive market; this market is open continuously at all $t \geq 0$. All firms observe the path of the exogenous state variable $X = \{X_s, 0 \leq s \leq \infty\}$. We let $\{\mathcal{F}_t\}_{t \geq 0}$ denote the filtration encoding the information observed by all firms prior to date $t$. In addition, a firm who currently owns a unit of capital privately observes its quality. The quality of each unit of capital is unobservable to all other firms. However, firms can observe to which sector the capital is currently allocated. For this reason, at each point in time $t$, there will be two prices in the market; one for capital currently located in sector A, denoted by $P_t^A$, and one for capital currently located in sector $B$, denoted by $P_t^B$.

Financial markets are complete with respect to the underlying probability space. In equilibrium, a complete financial market can be implemented with a risk-free asset and a market index. The state-price density $\xi$ (the price of Arrow-Debreu securities per unit of probability) evolves according to

$$\frac{d\xi_t}{\xi_t} = -r_t\, dt - \psi_t\, d\tilde{X}_t,$$

where $r_t$ is the risk-free rate of return and $\psi_t$ is the price of risk associated with unexpected

[7] Formally, there exists $a$, $A$ such that $0 < a < A < \infty$ and $\frac{\partial}{\partial q}\pi_i, \frac{\partial^2}{\partial q^2}\pi_i \in (a, A)$ for all $(i, q, x)$. Without imposing any structure on the distribution of capital quality, it is without loss to normalize $\pi_A(\theta, x_A) = \theta$. We have not done so here because at various points we will put additional structure on the production technology.

changes in the aggregate state ($d\tilde{X}_t \equiv dX_t - E_t[dX_t]$). We will require that the state-price density satisfy the transversality condition that $\lim_{t\to\infty} \xi_t = 0$. Following convention, we will refer to the growth in the state-price density as the stochastic discount factor (SDF).

*Firms.* There exists a mass $M > 1$ of competitive firms located in each sector. Firms maximize their market value by undertaking a capital allocation decision. Consider a sector $i$ firm who purchases a unit of capital at date $t$. Upon doing so, the firm will observe the capital quality, $\theta$, and operate the capital until it is no longer optimal to do so. The decision facing the firm is when to reallocate (i.e., sell) their existing capital. Let $V_t^i(\theta)$ denote the firm's value for the unit of capital. Given an ($\mathcal{F}_t$-adapted) price process, $P_t^i$, the firm's problem can be written as

$$V_t^i(\theta) = \sup_{\tau \geq t} E_t \left[ \frac{1}{\xi_t} \int_t^\tau e^{-\delta\,(s-t)} \xi_s \, \pi_i(\theta, X_s) ds + e^{-\delta\,(\tau - t)} \, \xi_\tau \, P_\tau^i \right]. \tag{10}$$

Last, there is a mass $\delta\, dt$ of new firms created each period. New firms optimally choose in which sector to operate. This specification will ensure that upon the arrival of any shock, the distribution of capital has full support in the sector from which it is being reallocated.

*Households.* There exist a continuum of identical households, indexed by $h \in [0, 1]$. The households problem is to choose a consumption process, $c^h = \{c_t^h : 0 \leq t \leq \infty\}$, that maximizes their lifetime utility,

$$\sup_c E_0 \left[ \int_0^\infty e^{-\beta\, t}\, u(c_t)\, dt \right], \tag{11}$$

subject to the budget constraint,

$$w_0 \geq E_0 \left[ \int_0^\infty \xi_t \, c_t \, dt \right]. \tag{12}$$

Here, $\beta > 0$ is their rate of time preference and $W_0$ is the value of their initial endowment. We assume that $u$ is a smooth, weakly concave function. We will focus on the case of risk-neutral households in Section 3. In Section 4, we incorporate risk aversion.

*Equilibrium Concept.* To rigourously define an equilibrium of the economy, we will need the following notation and definitions. Aggregate consumption is denoted by $C_t = \int c_t^h dh$. $T_t^i(\theta)$ denotes the *policy* of a firm in sector $i$ who acquires a unit of capital of quality $\theta$ at time $t$. The policy is *admissible* if it is both adapted to the filtration $\{\mathcal{F}_s\}_{s\geq 0}$ and weakly larger than $t$. $\Theta_t^i \equiv \{\theta : T_s^i(\theta) = t, s \leq t\}$ denotes the set of capital qualities sold at date $t$ from sector $i$. Finally, $F_t^i$ denotes the distribution of capital quality and $\theta_t^i \equiv \inf\{\theta : T_s^i(\theta) \geq t, s \leq t\}$

denotes the lowest quality of capital allocated to sector $i$ at date $t$.

**Definition 1.** *A* ***competitive equilibrium*** *of the decentralized economy consists of admissible policies, $T^i_t(\theta) : \Omega \to \mathbb{R}_+$ and $\mathcal{F}$-adapted consumption, price and state density processes $c^h, P^i, \xi : [0, \infty] \times \Omega \to \mathbb{R}$ such that for each $i \in \{A, B\}$, $t \geq 0$, $\theta \in \Theta$, $j \neq i$, $h \in [0, 1]$:*

1. *Firm's capital allocation decisions are optimal: $T^i_t(\theta)$ solves (10).*
2. *Household's consumption decisions are optimal: $c^h$ solves (11) subject to (12).*
3. *The market for the consumption good clears: $C_t = Y_t \equiv \sum_i \int \pi_i(\theta, x) dF^i_t(\theta)$.*
4. *The market for capital clears: if $\Theta^i_t = \emptyset$, $P^i_t \geq \inf\{V^j_t(\theta) : \theta \geq \theta^i_t\}$.*
5. *New firms make zero profit: if $\Theta^i_t \neq \emptyset$ then $P^i_t = \mathbb{E}\left[V^j_t(\theta) | \theta \in \Theta^i_t, \mathcal{F}_t\right]$,*

Conditions 1-3 are straightforward. Condition 4 requires that the price for a unit of capital in sector $i$ cannot be less than the lowest possible value for that unit of capital in sector $j$. If the price was strictly less, then all firms in sector $j$ would demand capital at that price and demand would exceed supply. Besides having a natural economic interpretation, this condition rules out trivial candidate equilibria, such as one in which prices are always very low and trade never takes place. Condition 5 is motivated by free entry and says that the price of capital at time $t$ must be equal to the expected value of the reallocated capital at time $t$, which implies a firm who purchases a unit of capital cannot make positive (or negative) expected profits.

We first establish several standard, but useful, properties.

**Lemma 2.1.** *In any competitive equilibrium, the state price density is proportional to the household's discounted marginal utility of consumption, $\xi_t \propto e^{-\beta t} u'(C_t)$.*

In addition, the *skimming* property must hold. That is, lower quality capital is reallocated sooner than higher quality capital.

**Lemma 2.2** (Skimming)**.** *In any competitive equilibrium, $T^i_t(\theta)$ is weakly increasing in $\theta$.*

The intuition is the same as in the motivating example; firms with lower quality capital are more anxious to sell their capital, because their outside option to wait is less valuable due to lower output in the interim.

For both tractability and ease of exposition, we conduct our analysis within the class of *symmetric economies*. In a symmetric economy, the output of a firm depends only on the quality of its capital and whether that capital is allocated efficiently (i.e., to the more productive sector given the current state).

**Definition 2** (Symmetric economies). *The economy is symmetric if there exists a pair of functions* $\{\bar{\pi}, \underline{\pi}\}$ *and scalar* $\lambda$ *such that* $\pi_i(\theta, x_i) = \bar{\pi}(\theta)$ *for* $i \in \{A, B\}$, $\pi_i(\theta, x_j) = \underline{\pi}(\theta)$, *and* $\lambda_{ij} = \lambda$ *for* $i \neq j$.

For the remainder of the paper, we will restrict attention to symmetric economies. It is straightforward, though more notationally cumbersome, to extend results to a setting in which the economy is not symmetric. A symmetric economy is fully described by $\Gamma \equiv \{\underline{\pi}, \bar{\pi}, u, \beta, \delta, \lambda, F\}$. We refer to the *production technology* as a pair of functions $\{\underline{\pi}, \bar{\pi}\} : \Theta \to \mathbb{R}$. Unless otherwise stated, we assume there is no ambiguity in which sector is most efficient.

**Assumption 2.3** (Gains from trade). *The production technology satisfies* $\bar{\pi}(\theta) > \underline{\pi}(\theta)$ *for all* $\theta < \underline{\theta}$.

This assumptions ensures that the market for capital does not completely breakdown (see Remark 3.2). We refer to the *efficient sector* at any given time $t$ as the sector in which output is given by $\bar{\pi}$ at date $t$ (i.e., $i$ such that $X_t = x_i$).

## 3 Equilibrium with Risk-Neutral Households

We begin by focusing on the setting with risk neutral households, $u(c) = c$. In this case, $\xi_t = e^{-\beta t}$ (Lemma 2.1) and the short-term interest rate is simply equal to household's impatience, $r_t = \beta$. With the state-price density pinned down, the natural extension of the equilibrium from Section 1 can be characterized by two functions. The first is $\tau(\theta)$, which represents how long it takes a $\theta$-unit of capital to be reallocated following a productivity shock (and provided that no other shocks arrive in the interim). The second is $\bar{V}(\theta)$, which is the (endogenous) value of an efficiently allocated unit of capital of quality $\theta$. As in Section 1, we will construct a *fully-revealing* equilibrium, which requires that $\tau$ is strictly increasing in $\theta$. Here again, it will sometimes be easier to use the inverse of $\tau$, which we denote by $\chi_t \equiv \tau^{-1}(t)$, which represents the quality of capital type that is reallocated a period of length $t$ after the most recent shock.

To formalize the connection to the equilibrium objects in Definition 1, let $m_t \equiv t - \sup\{s \leq t : x_{s^+} \neq x_{s^-}\}$ denote the amount of time that has elapsed since the last shock arrived.

**Definition 3.** *The firm strategies and capital prices that are* ***consistent*** *with* $(\tau, \bar{V})$ *are given by:*

$$T_t^i(\theta) = \inf\{s \geq t : m_s = \tau(\theta), x_s \neq x_i\} \tag{13}$$

$$P_t^i = \begin{cases} \bar{V}(\chi(m_t)) & \text{if } x_t \neq x_i \text{ and } m_t < \tau(\bar{\theta}) \\ \bar{V}(\underline{\theta}) & \text{otherwise} \end{cases} \tag{14}$$

The main result of this section is the following.

**Theorem 3.1.** *In a symmetric economy with risk-neutral households and strict gains from trade, there exists a unique $(\tau^*, V^*)$ such that the firm strategies and capital prices consistent with $(\tau^*, V^*)$ are part of a fully-revealing competitive equilibrium.*

To sketch the argument, we proceed with a heuristic construction of the equilibrium based on necessary conditions, which can be reduced to a single initial value problem. This initial value problem has a unique solution, which proves that a unique candidate exists. We then verify that these necessary conditions are also sufficient.

According to the candidate equilibrium, the value a firm derives from capital depends only on its quality if it is efficiently allocated. If it is inefficiently allocated, the value derived also depends the lowest quality of capital remaining in the inefficient sector (or equivalently, $m_t$). Let $\underline{V}(\theta, \chi)$ denote the value of an inefficiently allocated $\theta$-unit when the lowest remaining quality of capital in the inefficient sector is $\chi \leq \theta$. According to $(\tau, \bar{V})$, the firm waits until $\chi = \theta$ to trade. Therefore, the evolution of $\underline{V}$ for $\chi < \theta$ is given by

$$\beta \underline{V}(\theta, \chi) = \underline{\pi}(\theta) - \delta \underline{V}(\theta, \chi) + \lambda(\bar{V}(\theta) - \underline{V}(\theta, \chi)) + \frac{\partial}{\partial \chi} \underline{V}(\theta, \chi) \dot{\chi}_t \tag{15}$$

When $\theta = \chi$, a firm with a misallocated $\theta$-unit sells at a price equal to $V(\chi)$. We abuse notation by letting $P(\theta)$ denote the price at which a firm in the inefficient sector sells a $\theta$-unit to a firm in the efficient sector. Hence, a necessary boundary condition for $\underline{V}$ is given by

$$\underline{V}(\theta, \theta) = P(\theta). \tag{16}$$

The (local) optimality condition—required to ensure that firm optimality holds—is that when $\theta = \chi$, the firm with a $\theta$-unit is just indifferent between selling immediately and waiting an "instant". In other words, the firm's value function must smoothly paste to the path of prices.

$$P'(\chi) = \frac{\partial}{\partial \chi} \underline{V}(\theta, \chi) \bigg|_{\theta = \chi} \tag{17}$$

In order for the zero profit condition to hold, the price at which capital transacts must be equal to its value in the efficient sector. This requires that

$$P(\theta) = \bar{V}(\theta). \tag{18}$$

Evaluating (15) at $\theta = \chi_t$ using (16)-(18), we arrive at

$$\dot{\chi}_t = \frac{\rho \bar{V}(\chi_t) - \underline{\pi}(\chi_t)}{\bar{V}'(\chi_t)}, \tag{19}$$

where $\rho = \beta + \delta$, represents the firm's effective discount rate. Note that (19) is analogous to (5), where $\pi_B/r$ is replaced with $\bar{V}$. It is also worth noting that the rate at which productivity shocks arrive, $\lambda$, does not enter directly into (19). This is because the price the firm gets upon selling capital is equal to the value of that capital if another shock were to arrive (in which case the firm would retain possession). Nevertheless, $\lambda$ does play an important role in determining the equilibrium capital values and prices.

**Equilibrium Value of Capital**

Consider an arbitrary $(\tau, V)$ and note that the value of a unit of inefficiently allocated capital when $\chi = \underline{\theta}$ can be written as

$$\underline{V}(\theta, \underline{\theta}) = f(\tau(\theta))\frac{\underline{\pi}(\theta)}{\rho} + (1 - f(\tau(\theta))\bar{V}(\theta), \tag{20}$$

where

$$f(\tau) \equiv \int_0^{\tau} (1 - e^{-\rho t})\lambda e^{-\lambda t} dt + e^{-\lambda \tau}(1 - e^{-\rho \tau}), \tag{21}$$

denotes the expected discount factor until either (i) the state switches back, or (ii) the capital gets reallocated to the other sector. Similarly, the value of an efficiently allocated $\theta$-unit is given by

$$\bar{V}(\theta) = \frac{\rho}{\rho + \lambda}\frac{\bar{\pi}(\theta)}{\rho} + \frac{\lambda}{\rho + \lambda}\underline{V}(\theta, \underline{\theta}). \tag{22}$$

Solving (20) and (22) jointly, we arrive at

$$\bar{V}(\theta) = g(\tau(\theta))\frac{\underline{\pi}(\theta)}{\rho} + (1 - g(\tau(\theta)))\frac{\bar{\pi}(\theta)}{\rho}, \tag{23}$$

where $g(\tau) \equiv \frac{\lambda}{\rho + \lambda f(\tau)} f(\tau)$. The expression in (23) has an intuitive form. Capital spends some fraction of the time allocated efficiently and some fraction of the time misallocated. Therefore, its value is simply a weighted average of the value were it to be permanently efficiently allocated (i.e., $\frac{\bar{\pi}}{\rho}$) and permanently misallocated (i.e., $\frac{\underline{\pi}}{\rho}$). The amount the time it takes to get reallocated is determined by (19), which in turn depends on $V$; this illuminates the nature of the fixed point. The solution turns out to be quite tractable. By substituting $\chi_t$ for $\theta$ into (23) and substituting back into (19), we arrive at

$$\dot{\chi}_t = \frac{\rho\left(1 - g(t) + \frac{g'(t)}{\rho}\right)(\bar{\pi}(\chi) - \underline{\pi}(\chi))}{g(t)\underline{\pi}'(\chi) + (1 - g(t))\bar{\pi}'(\chi)}. \tag{24}$$

As before, the boundary condition is pinned down by the fact that the lowest type must reallocate immediately after the productivity shock and therefore

$$\chi_0 = \underline{\theta}. \tag{25}$$

The regularity conditions imposed on $\bar{\pi}$ and $\underline{\pi}$ ensure a unique solution exists and that this solution is monotonically increasing (see Lemma A.1). The last step in the proof of Theorem 3.1 is to verify that the candidate satisfies the remaining equilibrium conditions. The zero profit condition follows from the fact that capital of quality $\theta$ trades at a price of $V(\theta)$. Capital market clearing follows immediately from (14) and that $V(\theta)$ is equal to the value derived from a $\theta$-unit. Finally, in the appendix, we demonstrate that a firm who owns capital does not have a profitable deviation by showing that the Spence-Mirlees condition holds for firms' objective function, which verifies firm optimality.

**Remark 3.2** (Complete market breakdown)**.** *Equation (24) illustrates the importance of having strict gains from reallocating capital (Assumption 2.3) as it ensures that the numerator is strictly positive and thus $\chi$ and (hence $\tau$) are strictly increasing. On the other hand, if $\underline{\pi}(\theta) \geq \bar{\pi}(\theta)$ over some interval of $\Theta$, then in equilibrium, the market for used capital would breakdown completely and the reallocation process would get "stuck"; capital with quality in and above the interval would never be reallocated. Whether the reallocation process from one sector to another is completed in finite time also depends the gains from trade at the upper end of the distribution; if $\bar{\pi}(\overline{\theta}) > \underline{\pi}(\overline{\theta})$, then all capital gets reallocated in finite time, whereas if $\bar{\pi}(\overline{\theta}) = \underline{\pi}(\overline{\theta})$ then $\tau(\overline{\theta}) = \infty$.*

**Remark 3.3** (Rate of reallocation)**.** *Throughout the paper, we abuse terminology and refer to $\dot{\chi}_t$ as both the rate of skimming and the rate of reallocation. In general, the rate of reallocation also depends on the distribution of capital quality. That is, given the equilibrium rate of skimming through types, $\chi_t$, the rate of capital reallocation from sector $i$ to sector $j$ equals*

$$\frac{dk^j(t)}{dt} = \dot{\chi}_t \, dF^i(\chi_t). \tag{26}$$

*where $F_t^i$ is the cumulative distribution of capital quality in sector $i$ at time $t$.*

### 3.1 Reallocation following a Permanent shock

A special case of the model is when the productivity shock is *permanent*. To study the transition dynamics for this case, let $\lambda = 0$, assume that all capital is originally allocated to sector $A$, and the productivity shock occurs at $t = 0$ so that $B$ is the more productive sector for all $t \geq 0$.

This situation is effectively the same as that in Section 1: because sector $B$ is more productive, capital will transition from $A$ to $B$; due to adverse selection, the reallocation process occurs slowly over time. Since there are no further technological shocks, firms in sector $B$ retain the capital until it fully depreciates. Hence, firms have a value $\bar{\pi}(\theta)/\rho$ for a $\theta$-unit of capital. Thus, in any fully-revealing equilibrium, the rate at which at $\theta$-unit of capital is reallocated does not impact the price at which it trades.

**Proposition 3.4.** *Suppose that the productivity shock is permanent. Then, $g(t) = 0$ for all $t$ and (24) reduces to*

$$\dot{\chi}_t = \rho \frac{\bar{\pi}(\chi_t) - \underline{\pi}(\chi_t)}{\bar{\pi}'(\chi_t)}. \tag{27}$$

As expected, the expression for $\dot{\chi}$ in equation (27) is effectively the same as equation (5) in the example. Therefore, the equilibrium analyzed in the case of permanent productivity shocks is precisely the one characterized in Section 1. We revisit it here because it is useful for highlighting the economic environments under which various patterns in the rate of reallocation obtain. The numerator in (27) measures the magnitude of the productivity gains from reallocation as they depend on the quality of the capital; the larger the benefit of reallocation, the faster it takes place. The denominator measures the marginal productivity of capital quality in the efficient sector. It is perhaps surprising that higher marginal productivity of quality leads to *slower* reallocation. The intuition for this comes from the indifference condition of the cutoff type. Recall that the total change in prices with respect to time is given by

$$dP_t = \frac{\bar{\pi}'(\chi)}{\rho} \cdot \dot{\chi}_t dt.$$

Fixing $\dot{\chi}_t$, increasing the marginal productivity of capital quality increases the rate at which prices increase over time. In order for the cutoff type to remain indifferent, the reallocation rate must decrease. Using (27) and noting that $\dot{\chi}_t$ is strictly positive, we have the following result.

**Proposition 3.5.** *Suppose that the productivity shock is permanent. Then, the equilibrium rate of reallocation will increase (decrease) over time until all capital has been reallocated if and only if $(\bar{\pi} - \underline{\pi})/\bar{\pi}'$ is increasing (decreasing) over $\theta \in \Theta$.*

### 3.2 Reallocation with Transitory shocks

Here we examine the implications of transitory sectoral productivity shocks. In this case, firms investing in capital today will become sellers of capital at some point in the future. Therefore, in considering their willingness to pay for a $\theta$-unit of capital, firms must account for the potential costs associated with reallocation in the future.

We first explore the implications for the market price of capital. Recall that when the shock is permanent, the price at which a $\theta$-unit trades is equal to $\bar{\pi}(\theta)/\rho$, which is the present value of the future output that it generates for a firm in the efficient sector. With transitory shocks, this is no longer the case. Instead, the market price of capital includes an endogenous illiquidity discount.

**Proposition 3.6.** *If the productivity shock is transitory ($\lambda > 0$), the price at which a $\theta$-unit of capital trades is strictly less than $\bar{\pi}(\theta)/\rho$ for all $\theta > \underline{\theta}$.*

Figure 3 plots the equilibrium price at which capital trades as it depends on its quality and $\lambda$. As $\lambda$ increases, the overall value and price of capital decreases. The discount can be measured by the difference between the full information price and the price at which capital sells when firms are privately informed, i.e., $\frac{\bar{\pi}(\theta)}{\rho} - \bar{V}(\theta)$. Notice that the size of the discount depends on capital quality; since higher quality capital takes longer to be reallocated, it is associated with a larger discount.

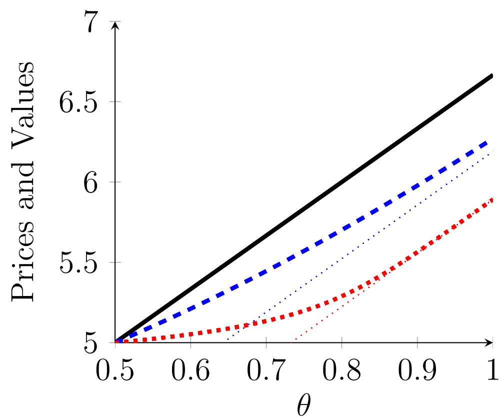



**Figure 3:** The effect of transitory shocks on the price of capital. The dashed blue corresponds to $\lambda = 0.1$ and the dotted red lines corresponds $\lambda = 1$. The black line represents the case when the shock is permanent ($\lambda = 0$), which also corresponds to the fully efficient value of capital. The fainter blue (red) dotted lines represent the hypothetical value of a unit of capital if it is never reallocated for $\lambda = 0.1$ ($\lambda = 1$), which approaches for $\lambda = 1$ and $\theta$ large. The figure uses CES production technology with $\alpha = 1$.

Next, we turn to the implications for the equilibrium rate of reallocation. More specifically, how does the effect on prices, driven by the transitory nature of shocks, impact the reallocation decision of firms? Recalling equation (3.4), intuition might suggest that, since the presence of a discount reduces the gains from trade, the rate of reallocation should decrease with $\lambda$. Indeed, this force is at play and will tend to slow down the rate at which reallocation occurs.[8]

[8]This is akin to what might one expect in a standard model with exogenous reallocation costs, in which

However, the intuition is incomplete because there is a second force at play: the nature of the illiquidity discount also affects sellers' incentives to delay in order to mimic higher types. Because the discount is larger for higher types – since their capital is endogenously less liquid – firms with low-quality capital have less incentives to delay in order to get a better price. Effectively, as $\lambda$ increases, equilibrium prices become less sensitive to capital quality, which mitigates the severity of the adverse selection problem and tends to increase the rate of reallocation. The next proposition formalizes this result.

**Proposition 3.7.** *Consider any two symmetric economies $\Gamma_x$ and $\Gamma_y$, which are identical except that $\lambda_x < \lambda_y$. There exists a $\bar{t} > 0$ such that the rate of reallocation is strictly higher in $\Gamma_y$ than in $\Gamma_x$ prior to $\bar{t}$, i.e., $\chi'_y(t) > \chi'_x(t)$ for all $t \in [0, \bar{t}]$.*

Formally, the argument goes as follows. Recall that the lowest-quality capital is always efficiently allocated and therefore $\rho V(\underline{\theta}) = \pi_1(\underline{\theta})$ in both $\Gamma_x$ and $\Gamma_y$. Fixing the equilibrium strategies from $\Gamma_x$, consider the effect of an increase from $\lambda_x$ to $\lambda_y$. Since the state is now switching more frequently and the rate of reallocation remains unchanged, firms with capital of quality $\theta > \underline{\theta}$ endure more misallocation, which gives them more incentive to imitate the lowest type. Now recall that by construction, types arbitrarily close to $\underline{\theta}$ were indifferent in $\Gamma_x$ between accepting $P(\underline{\theta})$ or waiting an instant. Hence, the increase in $\lambda$ will cause these types to strictly prefer to imitate $\underline{\theta}$. To restore the equilibrium in $\Gamma_y$, types near $\underline{\theta}$ must trade faster and the reallocation of capital increases, as we see in Figure 4. For higher $\theta$, the first effect (i.e., the reduction in the gains from reallocating) is larger and the rate of reallocation may increase or decrease.

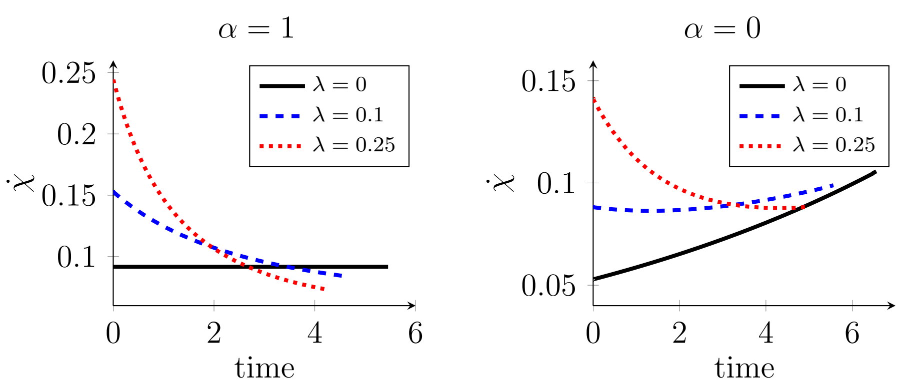



**Figure 4:** Equilibrium reallocation with transitory shocks and CES production technology for $\alpha = 1$ (left) and $\alpha = 0$ (right). The other parameters used are $\beta = 0.45$, $r = 0.15$, $z_A = \frac{1}{2}$, $z_B = 1$, $\Theta = [0.5, 1]$.

increasing the volatility of sectorial shocks typically leads to more delay as the option value of waiting increases.

Figure 4 also illustrates how the transitory nature of shocks affects the reallocation dynamics. In particular, the transitory nature of shocks tends to make $\dot{\chi}_t$, decreasing offsetting the effects of complementarity between quality and productivity ($\alpha = 0$) which generates an increasing rate of reallocation when $\lambda = 0$.

### 3.3 Response to a sectoral productivity shock

Next, we examine the response of aggregate quantities – output and productivity – to a sectoral productivity shock. The output of sector $i$ at time $t$ depends on the current distribution of project quality in that sector

$$Y_t^i = \int y_t^i(\theta)\, dF_t^i(\theta), \tag{28}$$

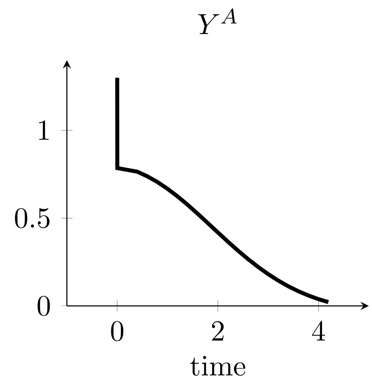
where $y_t^i(\theta)$ denotes the output of a unit of capital of quality $q$ in sector $i$ at time $t$. Aggregate output is then equal to $Y_t = Y_t^A + Y_t^B$. We compute the average productivity of capital in each sector as

$$X_t^i = \frac{Y_t^i}{k_t^i}. \tag{29}$$

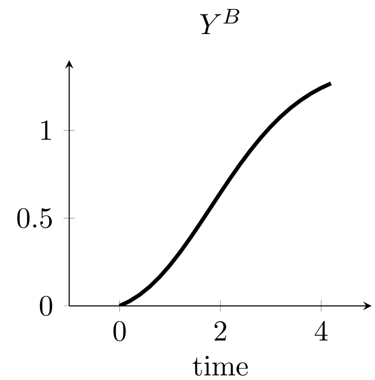
Since aggregate capital is constant, aggregate productivity is equal to total output, $X_t = Y_t$. We focus on the case where the gains from trade are constant, $\alpha = 1$, and the overall distribution of quality is distributed as a truncated normal on $\Theta$. We show the results in Figure 5.

Recall that a productivity shock causes the sectoral productivity of $A$ to fall and of $B$ to rise. Since all capital is initially allocated in sector $A$, aggregate output falls on impact, as we see in Panel (c). As the economy reallocates capital, output in sector $A$ continues to fall while output in sector $B$ rises. Once all capital is reallocated from sector $A$ to sector $B$, total output is restored to the pre-shock level. In this model, the response of output to a sectoral shock is qualitatively similar to that of a model with adjustment costs. However, the behavior of total factor productivity exhibits dynamics that are markedly different to a model with adjustment costs. In particular, Panels (d) and (e) show that productivity rises over time in *both* the sector from which capital exits (A) and in the sector to which it is being reallocated (B). In contrast, in the standard adjustment cost models, productivity would either be flat or display opposite patterns in each sector.[9]

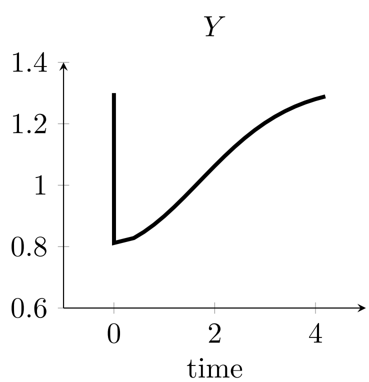
[9]Specifically, with constant returns average productivity of capital would be flat. With decreasing returns, productivity in sector A would increase while average productivity in sector B would decrease. Increasing returns to scale would generate the opposite pattern.



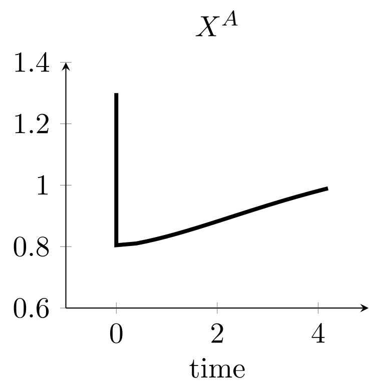

(a) output in sector A



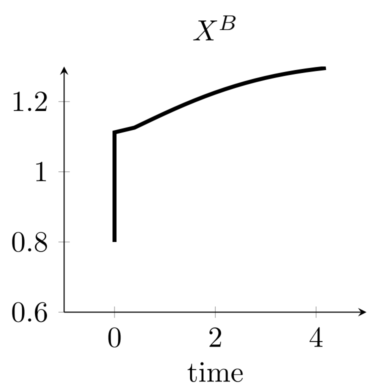

(b) output in sector B



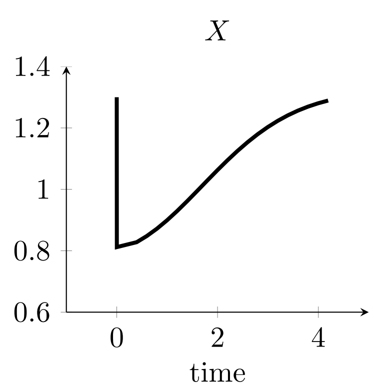

(c) total output




(d) productivity in sector A




(e) productivity in sector B




(f) total productivity

**Figure 5:** Response to a sectoral productivity shock, where at $t = 0$, sector B becomes the more productive sector. The distribution of quality $F(q)$ is beta in $\underline{\theta}$ and $\overline{\theta}$ with shape parameters $a = b = 2$. The figures uses constant gains from trade $\alpha = 1$ and transitory shocks $\lambda = 1/10$.

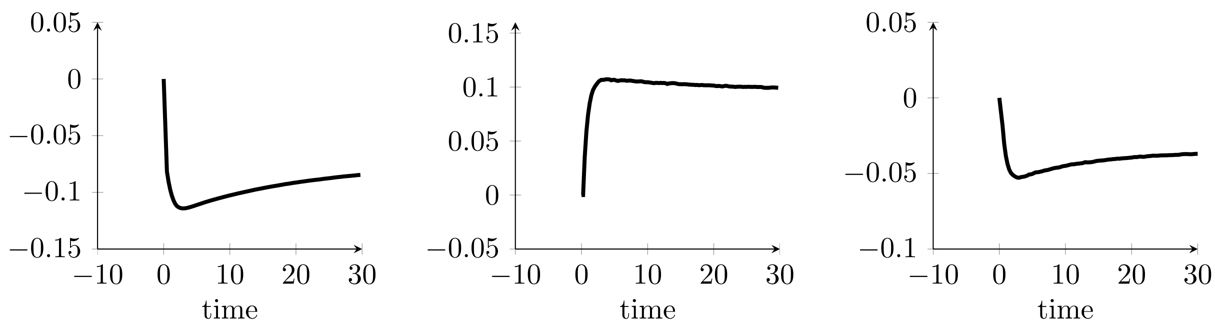
### 3.4 Response of the economy to unanticipated structural shifts

Next, we consider the effect of an unanticipated change in the model's structural parameters on aggregate output, the level of misallocation, and the rate of capital reallocation. Recent work has recently argued that shocks to reallocation costs can be useful for explaining features of the data (Eisfeldt and Rampini, 2006). This exercise allows us to interpret these exogenous reallocation shocks as a shift in the structural parameters of our model. We focus on the stationary model described in Section 3.2. To understand the impact of these shocks on output and productivity, we compute the level of misallocation at time $t$ as the percent of total potential output lost due to misallocation of capital, $1 - Y_t/\bar{Y}$, where $\bar{Y} = \int \bar{\pi}(\theta) dF(\theta)$ is the level of output in an economy without the adverse selection friction.

We consider two types of unanticipated parameter changes. First, we examine an increase in the dispersion of capital quality $\overline{\theta} - \underline{\theta}$. Second, we consider the effect of a change to the interest rate $r$. We compare the path of aggregate quantities as the economy transitions from the old to the new steady state.[10]

#### 3.4.1 Increase in the dispersion of capital quality

First, we consider an unanticipated increase in the dispersion of capital quality. We model this as an expansion in the support of the quality distribution of *new* capital inflows, holding the mean quality constant. The quality of the existing capital stock is unaffected.




(a) $\Delta R_t$ (b) $\Delta M_t$ (c) $\Delta \log(Y_t)$

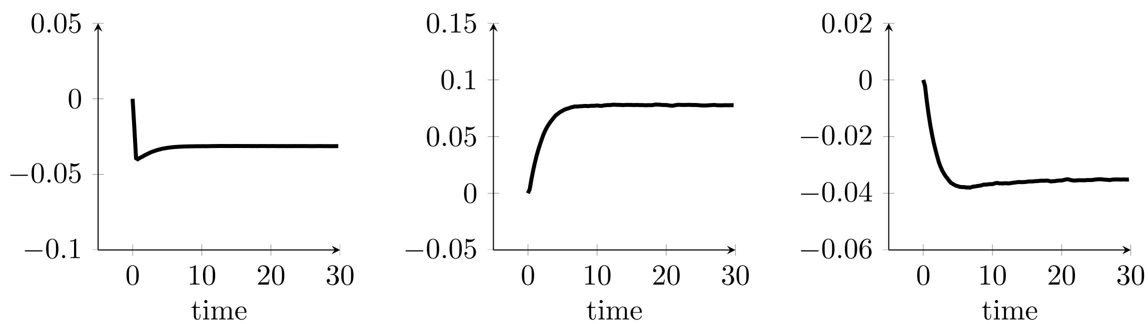
**Figure 6:** Response to an increase in the dispersion of capital quality. Figures plot mean difference from steady state across simulations.

[10]We construct impulse responses with respect to these structural changes as follows. We first simulate a sequence of sectoral productivity shocks assuming no structural shifts in parameters. Holding the sequence of sectoral productivity shocks fixed, we then permute the model by introducing an unanticipated parameter change at time 0 and compute the deviation across the two paths. We repeat this procedure 1,000,000 times and report mean deviations over all simulations.

Examining Figure 6 we see that, consistent with Eisfeldt and Rampini (2006), the model predicts that the dispersion in capital productivity is counter-cyclical, while the rate of reallocation is positively related to output growth. Increasing the dispersion of quality for new capital does not have a discrete effect upon impact, since new capital flows in slowly and is initially efficiently allocated. However, upon the arrival of the next productivity shock, the distribution of quality in the divesting sector is now greater. This increase in the degree of adverse selection implies that the rate of reallocation is slower. As buyers become more uncertain about capital quality, sellers need to wait longer to sell in order to signal their type. This decrease in the speed of transaction leads to a higher likelihood of capital misallocation, and therefore to lower aggregate output and productivity.

### 3.4.2 Reduction in the effective discount rate

Next, we analyze the impact of a reduction in the firm's effective discount rate $\rho$. In our setting, lowering the discount rate lowers the opportunity cost of delay for firms in the less productive sector (i.e., the left-hand side of equation (3)). To distinguish themselves from firms holding lower quality capital, firms with higher quality capital must wait even longer.




(a) $\Delta R_t$ (b) $\Delta M_t$ (c) $\Delta \log(Y_t)$

**Figure 7:** Response to a decrease in the interest rate.

As we see in Figure 7, lowering the discount rate leads to a slower rate of capital reallocation, more misallocation and thus lower productivity. The prediction that the rate of misallocation increases and output decreases with a reduction in the interest rate lies in sharp contrast to the prediction of models with exogenously specified costs of reallocation. In those models, lowering the rate at which agents discount the future increases the present value of the benefits from reallocating capital, which leads to faster reallocation and an increase in efficiency. If we interpret monetary policy as having an effect on firms' discount rate, clearly the comparative static in our model leads to different implications for how to stimulate reallocation compared to models in which reallocation costs are exogenously specified.

In sum, a reduction in the discount rate leads to a lower flow of output. However, the effect on present discounted values – such as economic efficiency or welfare – are ambiguous. Inefficiency, defined as the fraction of the discounted output lost due to misallocation, can increase or decrease with $\rho$ depending on parameter values and the fraction of misallocation capital. We should note that this result is in contrast to the implications of partial-equilibrium models with dynamic adverse selection (e.g., Janssen and Roy (2002); Fuchs and Skrzypacz (2013)) in which the length of inefficient delay is inversely proportional to the discount rate and hence a change in the discount rate does not effect overall efficiency. The difference is due to general equilibrium effects, specifically that the price at which capital trades includes an illiquidity discount.

# 4 Risk Averse Households

To this point, we have ignored general equilibrium effects on the interest rate by focusing on a setting with risk-neutral households. Let us now suppose that households exhibit CRRA utility: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. From Lemma 2.1, this implies that the state price density is given by

$$\xi_t = \exp(-\beta t) C_t^{-\gamma}.$$

The crucial difference here is that the SDF will depend on total output and therefore the distribution of capital. Therefore, the equilibrium rate of reallocation will depend on the distribution of capital and must be jointly determined with the SDF. In contrast, with risk-neutral households, $\dot{\chi}_t$ is independent of the distribution of capital.

We will start by studying how the desire to smooth consumption over time affects reallocation dynamics in response to a permanent productivity shock. We illustrate how our results from the previous sections can be extended and highlight two novel general equilibrium effects. First, the desire to smooth consumption increases the cost of delay and translates into faster reallocation. Second, the model predicts that large downturns are followed by fast recoveries whereas smaller negative shocks are followed by slower recoveries. Both of these predictions are in contrast to convex adjustment cost models in which the opposite prediction obtains.

We then re-incorporate aggregate risk into the economy with multiple transitory shocks. With sufficiently risk averse households, some capital remains misallocated despite the fact that output would increase by reallocating it. That is, the rate of reallocation reaches zero prior to all capital being reallocated. The intuition is that misallocated capital can serve as a hedge against a subsequent productivity shock. Thus, informational frictions not only generates delays in reallocation but can halt the reallocation process entirely.

### 4.1 Permanent Shocks

Suppose that at $t < 0$, both sectors are equally productive. At $t = 0$, a productivity shock arrives that makes sector $B$ relatively more productive ($\pi_B > \pi_A$). Capital will then gradually flow from sector $A$ to sector $B$. Our interest will be in characterizing the equilibrium rate of reallocation and how it depends on $\gamma$ as well as the initial distribution of capital across sectors, which we allow to be arbitrarily distributed according to smooth, strictly positive density functions $f_A, f_B$ over $\Theta$. For simplicity ignore both depreciation and new investment by setting setting $\delta = 0$. The primary additional consideration here is that $\xi_t$ depends on output dynamics and hence the rate of reallocation, $\dot{\chi}_t$. Therefore, the equilibrium value of capital will be determined endogenously. In this way, the effect is similar to the case with risk-neutral households and transitory shocks. However, here the mechanism works through the discount rate whereas with transitory shocks, the endogeneity worked through the cash flow channel.

To see this, recall that $\chi_t$ denotes the lowest quality capital allocated to sector $A$ at time $t$ and therefore aggregate output and consumption can be written as

$$C_t = Y_t = \int_{\chi_t}^{\bar{\theta}} \pi_A(\theta) f^A(\theta) d\theta + \int_{\underline{\theta}}^{\chi_t} \pi_B(\theta) f^B(\theta) d\theta.$$

Hence consumption grows according to

$$dC_t = (\pi_B(\chi_t) - \pi_A(\chi_t)) f^A(\chi)\dot{\chi}_t dt,$$

and thus $\dot{\chi}_t$ enters into the evolution of $\xi_t$

$$\frac{d\xi_t}{\xi_t} = -(\beta + \gamma C_t^{-1}\Big(\pi_B(\chi_t) - \pi_A(\chi_t)\Big) f^A(\chi)\dot{\chi}_t dt,$$

and leads to a short-term interest rate that is given by

$$r(\chi_t) = \beta + \gamma C_t^{-1}\Big(\pi_B(\chi_t) - \pi_A(\chi_t)\Big) f^A(\chi_t)\dot{\chi}_t. \tag{30}$$

The value of an efficiently allocated $\theta$-unit of capital at time $t$ (i.e., in state $\chi_t$) can be written as

$$V(\theta, \chi_t) = \nu(\chi_t)\pi_B(\theta), \tag{31}$$

where $\nu(\chi_t)$ is simply the price of an perpetuity at time $t$. Using standard arguments, $\nu$

satisfies

$$r(\chi_t)\nu(\chi_t) = 1 + \dot{\chi}_t \nu'(\chi_t), \qquad \nu(\overline{\theta}) = \beta^{-1} \tag{32}$$

and the zero-profit condition requires that

$$P_t = \nu(\chi_t)\pi_B(\chi_t). \tag{33}$$

For a fixed $\dot{\chi}_t$, we have now fully characterized the equilibrium price. The next phase of the analysis follows closely that in Section **??**. That is, we take the price as given and derive necessary conditions on $\dot{\chi}_t$. Analogous to (17), the optimality conditions for firms requires their value function smoothly pastes to prices. Letting $\underline{V}(\theta, \chi_t)$ denote the value of an inefficiently allocated unit of capital, the (local) optimality condition requires that

$$\frac{d}{dt}\underline{V}(\theta, \chi_t)\bigg|_{\theta=\chi_t} = \frac{d}{dt}P_t \tag{34}$$

and value matching requires that

$$\underline{V}(\chi_t, \chi_t) = P_t. \tag{35}$$

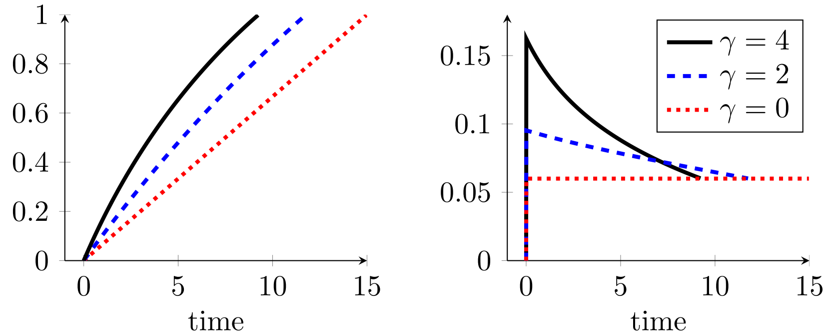
Using the law of motion for $\bar{V}$ and $\underline{V}$ along with (33)-(35), one arrives at

$$\dot{\chi}_t = \frac{1}{\nu(\chi_t)} \frac{\pi_B(\chi_t) - \pi_A(\chi_t)}{\pi'_B(\chi_t)}, \qquad \chi_0 = \underline{\theta}. \tag{36}$$

We are left with a pair of initial boundary problems (i.e., (32) and (36)) to which the (unique) fixed point characterizes the equilibrium.

**Theorem 4.1.** *In any economy in which households have CRRA utility and the productivity shock is permanent, there exists a unique* $(\tau^{**}, V^{**})$ *such that the firm strategies and capital prices consistent with* $(\tau^{**}, V^{**})$ *are part of a fully-revealing competitive equilibrium.*

The effect of consumption-smoothing motives on equilibrium reallocation is illustrated in Figure 8. The higher is $\gamma$, the stronger is the desire to smooth consumption. This increases the short-term interest rate, which makes it more costly for firms to delay reallocation and, in turn, speeds up the reallocation process. Note that increasing $\gamma$ will have the same qualitative implications for reallocation dynamics as a *reduction* the marginal adjustment cost.

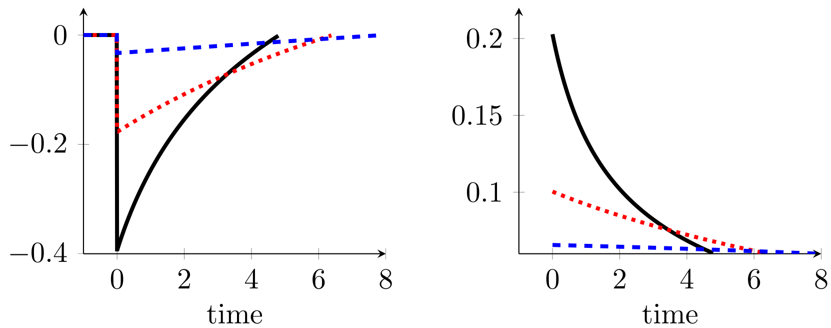



(a) Fraction of capital reallocated (b) Rate of reallocation

**Figure 8:** This figure illustrates how the reallocation dynamics depend on $\gamma$ for the case of a permanent productivity shock.

Another novel feature of the general equilibrium environment is that the rate at which the economy recovers from a productivity shock depends on the allocation of capital upon its arrival. To fix ideas, consider the case in which sector $A$ experiences a negative shock to productivity at $t = 0$. If all capital is initially allocated in Sector $A$ when the shock arrives, the economy will suffer a severe drop in output but the rate of reallocation will be high and the recovery process will be relatively quick. On the other hand, if capital is more evenly split across the two sectors when the shock arrives, then the drop in output will be smaller but the recovery process will be slower. The intuition is that when there is more capital to reallocate, the growth rate of consumption will be higher, which in return requires higher interest rates and lower $\nu$. This in turn raises the cost to firms in sector $A$ from delaying the sale of their capital and increases $\dot{\chi}_t$.[11] These dynamics are illustrated in Figure 9 for the case of a negative productivity shock to sector $A$.

[11] A similar comparative static prediction obtains with respect to either (i) fraction that sectors $A$ and $B$ constitute of the larger (unmodeled) economy and (ii) the magnitude of the productivity shock.




(a) Change in Output (b) Rate of reallocation

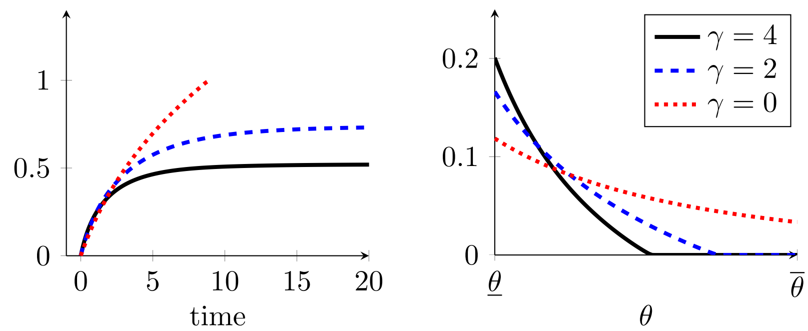
**Figure 9:** Recovery from a negative productivity shock to sector $A$ as it depends on the initial distribution of capital. The solid black line corresponds to the case where all of the capital is initially allocated in Sector $A$. The dotted red line (dashed blue line) correspond to the case where 50% (10%) of the capital is initially allocated to Sector $A$.

## 4.2 Transitory Shocks and Aggregate Risk

In the previous subsection, aggregate risk does not play a role in the reallocation decision of firms. The economic implications were driven by households' desire to smooth consumption over time. In this section, we explore how aggregate risk and households' desire to smooth consumption across aggregate states affects reallocation dynamics. We do so by extending the analysis from the previous subsection to a situation in which there are multiple transitory shocks. We focus attention on the reallocation dynamics from sector $A$ to sector $B$ when sector $B$ is currently more productive ($\pi_B > \pi_A$), but sector $A$ will become more productive at some (random) point in the future.

By using backward induction on the number of shocks yet to arrive, we derive the system of differential equations characterizing the reallocation dynamics in Appendix B. We solve this system numerically by applying standard techniques. Figure 10 illustrates an important finding from this exercise. Namely, that the rate of reallocation reaches zero prior to the all of the capital being reallocated to sector $B$. This implies that some capital remains *persistently* misallocated. The intuition is that misallocated capital can serve as a hedge against a subsequent productivity shock. Thus, informational frictions not only generates delays in reallocation but can halt the reallocation process entirely. In an adjustment cost model, this would correspond to an arbitrarily large adjustment cost beyond a certain threshold.

One might be tempted to consider a model in which shocks continue to arrive ad infinitum. We expect qualitatively similar results to obtain in such a model. However, analogous to macroeconomic models with heterogeneous households, solving for the equilibrium of such



(a) fraction reallocated (b) rate of reallocation

**Figure 10:** Reallocation dynamics in the presence of aggregate risk. This figure plots the dynamics prior to the arrival of the last shock.

a model requires keeping track the entire distribution of capital across sectors and thus an infinite dimensional state space. In order to overcome this problem, one would need to develop an approximate solution method (e.g., Krusell and Smith, 1998). Such an exercise may be useful to undertake in future work.

# 5 New Investment

To this point, we have focused on how adverse selection affects the reallocation of existing capital. Here, we illustrate how the mechanism can be incorporated into a model of new investment.

The economy has a mass of serial entrepreneurs and investors (or households). Output is generated by projects. Both investors and entrepreneurs can manage projects but only entrepreneurs have the ability to create new ones. Investment opportunities (or innovations) are indexed by $i$ and arrive randomly. Upon arrival, entrepreneurs can take advantage of the opportunity by investing I units of the consumption good to create a new project. Projects are heterogeneous in both their quality and the vintage of investment opportunity. A project using an innovation of vintage $i$ and quality $\theta$ produces a flow output $\pi_i(\theta)$. While the vintage of a project's innovation is observable, its quality is not.

Entrepreneurs have limited capacity or financial capital. Hence, in order to be able to take advantage of the new investment opportunity, they must sell their current project. However, as before, entrepreneurs are privately informed about the quality of their current project. It is natural to think that entrepreneurs may exhibit persistence in their ability to create projects. To capture this, with probability $\kappa \in [0, 1]$ the next project the entrepreneur creates is of the same quality, $\theta$, as her current project and with probability $(1 - \kappa)$ the quality of the new project is drawn from $F$ on $[\underline{\theta}, \overline{\theta}]$. For simplicity, the discount rate is fixed at $r$ and all projects have positive net-present value.

To illustrate how the adverse selection problem can lead to delays in new investment, assume all entrepreneurs are initially managing projects from innovation 0 and an investment opportunity (using innovation 1) arrives at $t = 0$. As long as an entrepreneur with the highest quality project is not willing to trade at the price for the average quality firm, not all entrepreneurs are willing to sell their firms immediately. As before, the equilibrium will have a gradual sale of gradual sale of firms in sector 0 to investors, and consequently, gradual investment in the new opportunity.

### 5.1 Equilibrium

The equilibrium construction follows steps similar to those in previous sections so we will omit formal details here. At the time of trade, the equilibrium reveals the type, hence there is only one type trading each instant. Investors are competitive, hence their break-even condition implies that, the price of a project of quality $\theta$ in sector $i = 0$ is $P(\theta) = \frac{\pi_0(\theta)}{r} > I$.

The next step involves determining the time, $\tau(\theta)$, that an entrepreneur owning a project of type $\theta$ will sell to investors. Once the entrepreneur has started his new firm of quality $\tilde{\theta}$, the firm will generate a profit flow of $\pi_1(\tilde{\theta})$ forever. Hence, his valuation of the new firm once it is created equals

$$\bar{V}(\tilde{\theta}) = \frac{\pi_1(\tilde{\theta})}{r}. \tag{37}$$

After the arrival of the innovation, but prior to the creation of a new project, the entrepreneur's expected payoff is equal to his conditional expectation of (37),

$$E_{\tilde{\theta}}\left[\bar{V}(\tilde{\theta})\right] = \kappa \bar{V}(\theta) + (1-\kappa)\int \bar{V}(\theta)\, dF(\theta), \tag{38}$$

discounted for the fact that the entrepreneur needs to wait until $\tau(\theta)$ to sell. Consequently, an entrepreneur of type $\theta$ has a continuation value that is a function of the lowest remaining entrepreneur in sector $i = 0$, denoted by $\chi$

$$V_0(\theta,\chi) = \frac{\pi_0(\theta)}{r} + e^{-r(\tau(\theta)-\tau(\chi))}\left(\kappa \bar{V}(\theta) + (1-\kappa)\int \bar{V}(\theta)\, dF(\theta) - I\right). \tag{39}$$

In equilibrium, the entrepreneur of type $\chi$ must be locally indifferent at the time of sale. This indifference condition can be written as

$$P'(\chi)\,\dot{\chi}_t = r\left(\kappa V_1(\theta) + (1-\kappa)\int \bar{V}(\theta) dF(\theta) - I\right). \tag{40}$$

Combining the two equations above yields a differential equation in $\chi$

$$\dot{\chi}_t = r\,\frac{\kappa\,\pi_1(\chi_t) + (1-\kappa)\int \pi_1(\theta)dF(\theta) - I}{\pi_0'(\chi_t)} \tag{41}$$

Equation (41), along with the boundary condition that $\chi(0) = \underline{\theta}$, pins down the unique equilibrium.

The persistence parameter $\kappa$ plays an interesting role in the rate of new investment. If $\kappa = 0$, the quality of the new projects that entrepreneurs create is independent of the quality of their current project. In this case, the entrepreneurs' expected return from investing in the new technology (i.e., the numerator in (41)) is independent of their type $\theta$. However, if their type is persistent, $\kappa > 0$, then the gains from trade is increasing in their type $\theta$. In this case the entrepreneurial talent and the new investment opportunity are complements, and the degree of complementarity increases with $\kappa$. Recalling our discussion of the case where $\alpha < 1$ in Proposition 1.1, when gains from trade are increasing in quality, trade is slower with the low types and then speeds up with higher types, or equivalently the rate of investment in new projects $\dot{\chi}_t$ is increasing over time. The same is true here even if the productivity improvement associated with the innovation, $\pi_1(\theta) - \pi_0(\theta)$, are independent of quality $\theta$.

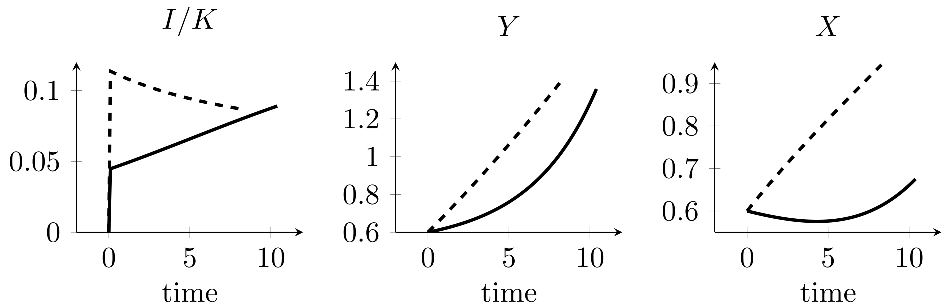
### 5.2 Output and productivity

Next, we analyze the model's implications for the dynamics of output and TFP of each technology and in the economy as a whole, defined as in (28) and (29) respectively.

Once the innovation becomes available and entrepreneurs start creating new projects, total factor productivity in the new sector is slowly increasing over time. This gradual increase in productivity occurs as progressively more talented entrepreneurs sell their old projects and create projects using the new innovation. However, once the new technology becomes available, aggregate TFP can actually *decrease*. This productivity drop can occur because, even though the new sector maybe on *average* more productive, the first projects created using the new technology sector are of below average quality – since they are created by below-average entrepreneurs. The higher the persistence, the greater the drop in measured TFP. The following proposition states the conditions under which this drop in productivity occurs.

**Proposition 5.1.** *Upon the arrival of an innovation, economy wide TFP is initially decreasing (and eventually increasing) over time if and only if* $\kappa\pi_1(\underline{\theta}) + (1-\kappa)\,E_{\tilde{\theta}}[\pi_1(\tilde{\theta})] < E[\pi_0(\theta)]$.

Furthermore, the total magnitude of the TFP drop will be higher the greater the persistence in quality $\kappa$. As we see in Figure 11, when entrepreneurial talent is more transferable to the new technology (high $\kappa$), the process of creating new projects is further delayed; investment responds with a lag, and aggregate productivity dips on impact. The possibility that measured total factor productivity might drop at the onset of the arrival of an innovation (as in Figure 11(c)) is consistent with several empirical studies (David, 1990; Jovanovic and Rousseau, 2005).




(a) Rate of new investment

(b) Aggregate output

(c) Aggregate TFP

**Figure 11:** Aggregate output, productivity and rate of new investment. A solid line represents the high persistence case ($\kappa = 0.75$) and the dotted line represents lower persistence ($\kappa = 0.25$)

# 6 Empirical Evidence

Here, we discuss our model's relation to the data.

## 6.1 An Empirical Test

The fact that the model's predictions pertain to unobservable characteristics makes developing direct tests of the mechanism inherently challenging. In our empirical design, we exploit the fact that the unobservable quality of capital in our model is correlated with ex-post measures of profitability.

We focus on the change in ownership from entrepreneurs to investors following a firm's initial public offerings (IPOs).[12] IPOs offer an attractive setting to test our mechanism for two reasons. First, the amount of public information available about the firm is scarce prior to its IPO and hence an informational asymmetry between sellers (the entrepreneur) and buyers (investors) seems quite plausible. Second, IPOs are a setting in which we have available ex-post measures of operating performance for the asset being traded. Hence, even

[12] In the previous section, entrepreneurs sold their projects in order to create new ones, though clearly there are other economic reasons for these transactions. For example, risk aversion on the part of entrepreneurs can be viewed as a higher flow operating cost while the firm is private and provides a motive for diversification. Consumptions smoothing motives and expansionary investment are other natural explanations.

though firm quality is unobservable, it is likely to be correlated with post-IPO measures of the firm's profitability. We focus on testing the following two implications of our mechanism.

**Prediction 1:** *Controlling for observable characteristics, entrepreneurs with more profitable firms wait longer to IPO. Therefore, the length of time to IPO should be positively correlated with post-IPO measures of profitability.*

**Prediction 2:** *In a fully revealing equilibrium, the market correctly interprets delay as a signal of profitability and prices adjust accordingly. Therefore, the length of time to IPO should* ***not*** *be correlated with post-IPO stock returns.* We use the length of time elapsed

between a firm's incorporation and its IPO as a proxy for how long entrepreneurs wait to IPO and control for observable characteristics at the time of the IPO. These characteristics include firm size and profitability at the time of the IPO, along with IPO-year dummies, or IPO year interacted with industry dummies. We relegate all details of our empirical specification in Appendix B. We present out findings in Table 2.

Examining Panel A, we see that the length of time from a firm's incorporation to its IPO is predictive of its future profitability (return on assets, or ROA) at horizons of up to 5 years. This predictive relation is robust to controlling for observable characteristics, including controls for firm size and its profitability at the time of the IPO, as well as IPO-year dummies, or IPO year interacted with industry dummies. Columns (I) to (III) present results with different controls. The economic magnitudes are substantial. Focus on column (III), which compares two firms that did an IPO at the same time, belong in the same industry, and have the same size and profitability at the year of the IPO. The firm that belongs in the 75-th percentile in terms of the age at IPO experiences a 4.4% to 9.8% higher ROA than the firm at the 25-th percentile over a one to five-year horizon. For comparison, the interquartile range in firm ROA ranges from 18.6% to 24% over a one to five-year horizon. This finding supports our model's prediction that entrepreneurs with higher quality capital delay the sale of their capital for longer as a signal of quality to the market.

Importantly, as we see in Panel B of Table 2, even though a firm's age predicts future profitability, it *does not* predict its stock returns following the IPO decision. This lack of return predictability, which is common across all specifications – see Columns (I) and (III) in Panel B – implies that the higher ex-post profitability associated with older firms does not represent news to the market. This finding suggests that, consistent with our model, the firm's price at the time of the IPO is fully revealing of its quality.

In sum, our empirical results are supportive of our mechanism. Naturally, these results come with the usual disclaimer in that they are based on correlations and we do not establish causality. There exist several other theories that make predictions about the timing of the

IPO decision (an incomplete list includes Maksimovic and Pichler, 2001; Pastor and Veronesi, 2005; Pastor, Taylor and Veronesi, 2009). However, to the best of our knowledge, none of these theories make explicit predictions about the timing of the decision and the firm's age. An exhaustive empirical analysis that establishes a causal link and allows us to distinguish between alternative theories is left for future work.

### 6.2 Additional Supporting Evidence

In addition to the empirical test conducted above, our model's predictions are also consistent with a variety of indirect evidence in the existing literature.

In the context of reallocation of human capital, Wagner and Zwick (2012) exploit data from the German apprenticeship system to document the role of adverse selection. Consistent with our model, they find that workers who miqrate to new firms quickly after completing their apprenticeship (i.e., "early switchers") earn lower wages and are less productive than workers who stay with their existing firms. In the reallocation of physical capital, Ramey and Shapiro (2001) document the following stylized facts that are consistent with our model: i) capital sells at a substantial discount relative to its replacement cost; ii) this discount is smaller if capital sells to other aerospace firms, which presumably have better ability to evaluate its quality; iii) the process of selling used equipment is lengthy.

Our model implies that an increase in the degree of adverse selection – for instance, an increase in the dispersion in capital quality – increases the cost of reallocation leading to lower output growth. This prediction is in line with Eisfeldt and Rampini (2006), who document that in recessions, the dispersion in capital productivity is higher while the rate of capital reallocation is lower. More generally, an increase in the degree of adverse selection can be interpreted as a reduction in the efficiency of financial markets due to an increase in the degree of information asymmetry across investors. Under this interpretation, our results shed some light on the behavior of the economy during the financial crisis of 2008. During the crisis, several asset markets experienced a marked drop in transactions.[13] This drop in capital liquidity would certainly have an adverse effect in the rate of capital misallocation. Indeed, there is some evidence that, in general, financial crises are accompanied with an increase in misallocation of resources (see, for instance Oberfield, 2013; Ziebarth, 2013). While the exact causes and consequences of financial crises are not yet fully understood, adverse selection appears to be an important component.

Our model also provides an economic explanation for why disinvestment should be more

[13] For instance, the IPO market essentially froze during the crisis. In 2008, there were only 31 IPOs – the lowest annual total since 1975 – with gross proceeds of $24.1 billion, of which $17.9 billion came from a single offering (source: WilmerHale 2009 IPO Report).

costly than investment, a prominent feature in structural models of investment (see e.g., Abel and Eberly, 1994, 1996). In particular, disinvestment involves the sale of used capital, where one naturally expects the adverse selection problem to be more severe, whereas investment often involves purchasing capital directly from its producers where the information friction is likely to be less severe (e.g., due to the reputational concerns of producers). Along these lines, Cooper and Haltiwanger (2006) estimate a structural model of convex and non-convex adjustment costs using plant-level data. Their estimates imply a substantial spread between the purchase and sale price of capital.

Clearly, adverse selection is not the only mechanism inhibiting the efficient allocation of capital. Nor is it the only way to rationalize these patterns. The existing literature is rich with explanations. Physical (convex) costs, search, financial frictions, learning, time-to-build and other factors are likely to be important components in the allocation of new and existing capital. Indeed, one key benefit of the adjustment cost approach is to absorb a variety of frictions into a single cost function. We have abstracted away from these considerations in order to highlight the key ideas of the paper. Incorporating these frictions in macro-economic models that are suitable for calibration – and therefore providing a way to quantitatively asses the importance of these frictions – is a promising path for future work. We view our work as an important step in this direction.

# 7 Conclusion

In this paper, we have incorporated persistent adverse selection into a competitive decentralized economy to study the dynamics of capital allocation and new investment. The information friction leads to slow movements in capital reallocation, lagged investment following technological innovations, and provides a micro-foundation for convex adjustment cost models. The model generates a rich set of dynamics for aggregate quantities.

## References


**Abel, Andrew B**, "Optimal Investment under Uncertainty," *American Economic Review*, March 1983, *73* (1), 228–33.

_ **and Janice C Eberly**, "A Unified Model of Investment under Uncertainty," *American Economic Review*, December 1994, *84* (5), 1369–84.

**Abel, Andrew B. and Janice C Eberly**, "Optimal Investment with Costly Reversibility," *Review of Economic Studies*, 1996, *63(4)*, 581–93.

**Akerlof, George**, "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 1970, *84*, 488–500.

**Banerjee, Abhijit V. and Esther Duflo**, "Growth Theory through the Lens of Development Economics," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, 2005, chapter 7, pp. 473–552.

**Bigio, Saki**, "Endogenous Liquidity and the Business Cycle," *Working Paper*, 2013.

**Brennan, Michael J and Alan Kraus**, "Efficient Financing under Asymmetric Information," *Journal of Finance*, December 1987, *42* (5), 1225–43.

**Chang, Brianna**, "Adverse Selection and Liquidity Distortion in Decentralized Markets," *working Paper*, 2012.

**Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans**, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1–45.

**Cooper, Russell W. and John C. Haltiwanger**, "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 2006, *73* (3), 611–633.

**Daley, Brendan and Brett Green**, "Waiting for News in the Market for Lemons," *Econometrica*, 2012, *80(4)*, 14331504.

_ **and** _ , "An Information-based Theory of Time-varying Liquidity," *Working Paper*, 2014.

**David, Paul A.**, "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox," *The American Economic Review*, 1990, *80* (2), pp. 355–361.

**Eberly, Janice and Neng Wang**, "Capital Reallocation and Growth," *American Economic Review*, May 2009, *99* (2), 560–66.

**Eisfeldt, Andrea L.**, "Endogenous Liquidity in Asset Markets," *Journal of Finance*, 02 2004, *59* (1), 1–30.

_ **and Adriano A. Rampini**, "Capital reallocation and liquidity," *Journal of Monetary Economics*, April 2006, *53* (3), 369–399.

_ **and** _ , "New or used? Investment with credit constraints," *Journal of Monetary Economics*, November 2007, *54* (8), 2656–2681.

**Finkelstein, Amy and James Poterba**, "Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market," *Journal of Political Economy*, February 2004, *112* (1), 183–208.

**Fuchs, William and Andy Skrzypacz**, "Costs and Benefits of Dynamic Trading in a Lemons Market," *Working Paper*, 2013.

**_ , Aniko Oëry, and Andy Skrzypacz**, "Transparency and Distressed Sales under Asymmetric Information," *Working Paper*, 2014.

**Fudenberg, Drew and Jean Tirole**, *Game Theory*, MIT Press, 1991.

**Gibbons, Robert and Lawrence F Katz**, "Layoffs and Lemons," *Journal of Labor Economics*, October 1991, *9* (4), 351–80.

**Guerrieri, Veronica and Robert Shimer**, "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," *American Economic Review*, 2014, *104* (7), 1875–1908.

**Hörner, Johannes and Nicolas Vieille**, "Public vs. Private Offers in the Market for Lemons," *Econometrica*, 2009, *77(1)*, 29–69.

**House, Christopher L. and John V. Leahy**, "An sS Model with Adverse Selection," *Journal of Political Economy*, June 2004, *112* (3), 581–614.

**Hsieh, Chang-Tai and Peter J. Klenow**, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.

**Janssen, Maarten C. W. and Santanu Roy**, "Dynamic Trading in a Durable Good Market with Asymmetric Information," *International Economic Review*, 2002, *43:1*, 257–82.

**Jovanovic, Boyan and Peter L. Rousseau**, "General Purpose Technologies," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, 2005, chapter 18, pp. 1181–1224.

**Korajczyk, Robert A, Deborah J Lucas, and Robert L McDonald**, "The Effect of Information Releases on the Pricing and Timing of Equity Issues," *Review of Financial Studies*, 1991, *4* (4), 685–708.

**Krusell, Per and Anthony Smith**, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 1998, *106(51)*, 867–896.

**Kurlat, Pablo**, "Lemons Markets and the Transmission of Aggregate Shocks," *American Economic Review*, September 2013, *103* (4), 1463–89.

**Leland, Hayne E and David H Pyle**, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *Journal of Finance*, May 1977, *32* (2), 371–87.

**Lizzeri, Alessandro and Igal Hendel**, "Adverse Selection in Durable Goods Markets," *American Economic Review*, December 1999, *89* (5), 1097–1115.

**Lucas, Deborah J and Robert L McDonald**, "Equity Issues and Stock Price Dynamics," *Journal of Finance*, September 1990, *45* (4), 1019–43.

**Maksimovic, V and P Pichler**, "Technological innovation and initial public offerings," *Review of Financial Studies*, 2001, *14* (2), 459–494.

**Michaely, R and WH Shaw**, "The pricing of initial public offerings: tests of adverse-selection and signaling theories," *Review of Financial Studies*, 1994, *7* (2), 279–319.

**Myers, Stewart and Nicholas Majluf**, "Corporate Financing and Investment Decisions when Firms have Information Investors do not have," *Journal of Financial Economics*, 1984, *13*, 187–221.

**Oberfield, Ezra**, "Productivity and Misallocation During a Crisis: Evidence from the Chilean Crisis of 1982," *Review of Economic Dynamics*, January 2013, *16* (1), 100–119.

**Pastor, LLubos and Pietro Veronesi**, "Rational IPO Waves," *The Journal of Finance*, 2005, *60* (4), 1713–1757.

_ **, Lucian A. Taylor, and Pietro Veronesi**, "Entrepreneurial Learning, the IPO Decision, and the Post-IPO Drop in Firm Profitability," *Review of Financial Studies*, 2009, *22* (8), 3005–3046.

**Ramey, Valerie A. and Matthew D. Shapiro**, "Displaced Capital: A Study of Aerospace Plant Closings," *Journal of Political Economy*, October 2001, *109* (5), 958–992.

**Wagner, Robert and Thomas Zwick**, "How acid are lemons? Adverse selection and signalling for skilled labour market entrants," *ZEW Discussion Papers, No. 12-014*, 2012.

**Zeidler, Eberhard**, *Nonlinear Functional Analysis and its Applications I: Fixed Point Theorems*, Springer, 1998.

**Ziebarth, Nicolas**, "Are China and India Backwards? Evidence from the 19th Century U.S. Census of Manufactures," *Review of Economic Dynamics*, 2013, *16* (1), 86–99.

# A Appendix

## A.1 Proofs

*Proof of Lemma 2.1.* Follows immediately from the households first order condition and the goods market clearing condition. □

*Proof of Lemma 2.2.* If the skimming property did not hold, then there exists $(i, t)$ such that $t_1 \equiv T_t^i(\theta) > t_0 \equiv T_t^i(\theta')$ for some $\theta' > \theta$. Since $q$ prefers to wait until $T_t^i(\theta)$ then $V_{t_0}^i(\theta) \geq P_{t_0}^i$. Since $\theta'$ accepts at $t_0$, $V_{t_0}^i(\theta) = P_{t_0}^i$. But since $\pi_i$ is increasing, $\theta'$ could do strictly better by mimicking the type $\theta$, which violates (10). □

The proof of Theorem 3.1, relies on Lemma A.1, which we prove below.

**Lemma A.1.** *There exists a unique $\chi^*$ that satisfies (24) and (25). Furthermore, $\chi^*$ is strictly increasing.*

*Proof.* Note first that (24)-(25) is an initial value problem of the form

$$\chi'(t) = f(t, \chi(t)), \quad \chi(0) = \underline{q} \tag{42}$$

To verify existence and uniqueness of a solution, we will apply the Picard-Lindelof Theorem (see Zeidler (1998), Theorem 3.A.) To do so, it is sufficient to verify several properties of $f$ : (i) $f(t, x)$ is continuous on $[0, T] \times [\underline{q}, \bar{q}]$; (ii) $f$ is bounded (iii) that $f(t, x)$ is Lipschitz. Property (i) is by inspection (since both $g$ and $\pi_i$ are continuously differentiable). Property (ii) follows immediately from the expression for $g$ and the conditions placed on $\pi_i$. To demonstrate (iii), it suffices to show that $\frac{d}{dx} f(t, x)$ is bounded, which follows from the restriction that $\pi_i$ have bounded first and second derivatives. □

*Proof of Theorem 3.1.* From Lemma A.1, there is a unique candidate (fully) revealing equilibrium. Thus, in order to prove the theorem, it suffices to check that the candidate satisfies the equilibrium conditions. The zero profit and capital market clearing conditions are satisfied by construction. To verify that firms optimize, note that no firm in the efficient sector strictly prefers to sell their capital since the price is $V(\underline{\theta})$, which is the least a firm can expect to earn by continuing to operate their capital. It remains to verify that there are no profitable deviations for firms in the inefficient sector. To see this, note that the sellers objective can be written as

$$u_\theta(t, P) = (1 - f(t)) \frac{\pi(\theta)}{\rho} + f(t) P$$

and therefore

$$\frac{\partial}{\partial \theta} \left( \frac{\partial u_\theta / \partial P}{\partial u_\theta / \partial t} \right) = \frac{f(\tau)}{f'(t)(P - \underline{\pi}(\theta)/\rho)} > 0,$$

which shows that the single-cross condition is satisfied. In this case, a standard result (Fudenberg and Tirole, 1991, chap. 7) is that the local IC constraint and monotonicity of $(\tau, P)$, which hold by construction, are sufficient to guarantee that no profitable global deviations exist. □

*Proof of Proposition 3.4.* That $g(t) = 0$ for $\lambda = 0$ is by inspection. That (24) then reduces to (27) follows immediately. □

*Proof of Proposition 3.5.* Taking the total derivative of the RHS of (27) with respect to time we get that

$$\chi''(t) = \rho \cdot \frac{d}{d\chi}\left(\frac{\bar{\pi}(\chi) - \underline{\pi}(\chi)}{\bar{\pi}'(\chi)}\right) \cdot \dot{\chi}_t$$

Since $\dot{\chi}_t(t) > 0$ for all $t \in [0, \tau(\bar{\theta})]$. The derivative of $\dot{\chi}_t$ with respect to time has the same sign as the derivative of $\frac{\bar{\pi}(\theta)-\underline{\pi}(\theta)}{\bar{\pi}'(\theta)}$ with respect to $\theta$. □

*Proof of Corollary 1.1.* Follows immediately from Proposition 3.5 and the fact that for CES production technology, $\frac{d}{d\theta}(\frac{\bar{\pi}-\underline{\pi}}{\bar{\pi}'})$ is strictly positive for $\alpha < 1$, strictly negative for $\alpha > 1$, and equal to zero for $\alpha = 1$. □

*Proof of Proposition 3.6.* This follows from (23) and the fact that (i) $\tau(\theta) > 0$ for all $\theta > \underline{\theta}$, and (ii) $g(t) > 0$ for all $t > 0$. □

*Proof of Proposition 3.7.* Using a subscript to represent elements of the relevant economy, we have that

$$\begin{aligned}\chi_2'(0) - \chi_1'(0) &= (1 - g(0;\lambda_2) + g(0;\lambda_2)) - (1 - g(0;\lambda_1) + g'(0;\lambda_1)) \\ &= g'(0;\lambda_2) - g_1'(0;\lambda_1) > 0\end{aligned}$$

Where the inequality follows from the fact that $\frac{d}{d\lambda}g'(0;\lambda) > 0$. Therefore, $\chi_2'(0) - \chi_1'(0) > 0$. By the continuity and boundedness of $\chi_1'$ and $\chi_2'$, there must exist $\bar{t} > 0$ such that the inequality holds for $t \in [0, \bar{t}]$. □

*Proof of Theorem 4.1.* The proof involves showing that there exists a unique candidate solution satisfying the joint system of differential equations and then verifying that the strategies and prices consistent with the candidate satisfy the equilibrium requirements.

Fix an economy, which can be represented by $\{f^A, f^B, \pi_A, \pi_B, \gamma, \beta\}$. Define

$$c(\chi) \equiv \int_{\chi}^{\bar{\theta}} \pi_A(\theta) f^A(\theta) d\theta + \int_{\underline{\theta}}^{\chi} \pi_B(\theta) f^B(\theta) d\theta.$$

Let $\tau, V$ denote an arbitrary candidate revealing equilibrium and note that the zero profit condition requires that $V(\theta,\chi) = \pi_B(\theta)\nu(\chi)$, therefore it is sufficient to characterize $\tau, \nu$. Assuming $\tau$ is strictly increasing and therefore invertible, define $\chi_t \equiv \tau^{-1}$ and $\phi(\theta) = \frac{1}{\tau'(\theta)}$. From (30), (32) and (36), we know that that any candidate revealing equilibrium must satisfy

$$\phi(\theta) = \frac{\pi_B(\theta) - \pi_A(\theta)}{\pi_B'(\theta)\nu(\theta)}, \qquad \tau(\underline{\theta}) = 0 \tag{43}$$

$$\left(\beta + \frac{\gamma}{c(\theta)}(\pi_B(\theta) - \pi_A(\theta))f^A(\theta)\phi(\theta)\right)\nu(\theta) = 1 + \phi(\theta)\nu'(\theta), \qquad \nu(\bar{\theta}) = \beta^{-1} \tag{44}$$

Substituting the ODE from (43) into (44), and rearranging, we arrive at an initial value problem of the form

$$\nu'(\theta) = f(\theta, \nu(\theta)), \quad \nu(\bar{\theta}) = \beta^{-1} \tag{45}$$

The proof of existence and uniqueness of a solution to (45) follows closely the proof of Lemma 3.1 and is therefore omitted. Letting $\nu^{**}$ denote this solution, substitute it into (43), and apply the same argument to get existence and uniqueness of $\tau^{**}$. The next step is to show that $\tau^{**}$ is strictly increasing. From equation (43), it suffices to show that $\nu^{**} > 0$ for all $\theta$. Suppose, to the contrary, that $\nu^{**}(\theta') < 0$ for some $\theta'$. Since $\nu^{**}$ is continuous and $\nu^{**}(\overline{\theta}) > 0$, there must exists a $\theta'' > \theta'$ such that $\nu^{**}(\theta'') = 0$, but this clearly violates (43). Therefore, $\tau^{**}$ is invertible. Let $\chi_t^{**}$ denote its inverse. Thus, we have shown there exists a unique candidate revealing equilibrium.

To verify the candidate is indeed part of an equilibrium, specify that $\xi_t = \exp(-\beta t)c(\chi_t^{**})^\gamma$ and $c_t^h = c(\chi_t^{**})$. Household optimality and market clearing of the consumption good is immediate. That the capital market clears and new firms make zero profit in the candidate follows immediately from the fact that only $\chi_t^{**}$ trades at time $t$ and the solution satisfies (33). Locally, firm optimality is by construction (i.e., (34)). That the firm's strategy is optimal globally follows from the same arguments as used in the proof of Theorem 3.1. □

*Proof of Proposition 5.1.* The expected productivity of a new firm is $\rho\bar{\pi}\left(\underline{\theta}\right) + (1-\rho)\, E_{\tilde{\theta}}\left[\bar{\pi}(\tilde{\theta})\right]$ and the average productivity when all firms are in the original sector is just the average productivity of the sector, $E\left[\underline{\pi}\left(\theta\right)\right]$. Hence, when the former is smaller than the latter the firms created upon arrival of new vintage are of below average $TFP$ and thus lower the average measured $TFP$ of the economy. □

## A.2 Transitory Shocks with Risk Averse Households

Suppose now that there are multiple transitory shocks and households are risk averse. Our analysis in Section 4.1 applies once the last shock arrives. Let us now consider the case in which there are two shocks. To fix ideas, suppose that at $t < 0$ all capital is allocated to sector $A$. At $t = 0$, a shock arrives that makes sector $B$ more productive. But this shift is not permanent: at some random time $\tau > 0$, another shock will arrive that will make sector $A$ the more productive sector. As before use $\bar{\pi}$ ($\underline{\pi}$) to denote the productivity of capital allocated efficiently (inefficiently).

One can think of the model as having two regimes. In the first regime ($t < \tau$), capital transitions from sector $A$ to sector $B$. In the second regime ($t > \tau$), capital transitions back to sector $A$. We use subscripts to denote to which regime the object refers. For example, $T_1(\theta)$ denotes the time at which a sector $A$ firm sells capital of quality $\theta$ to sector $B$ in the first regime. Let $\theta_1$ denote the lowest type remaining in sector A at the end of the first regime, i.e., $\theta_1 \equiv \inf\{\theta : T_1(\theta) > \tau\}$.

### A.2.1 Second Regime

We proceed by backward induction. Note that all $\theta > \theta_1$ are efficiently allocated at the beginning of the second regime. Hence, for all $t \geq \tau$:

$$\begin{aligned} Y_t &= \int_{\underline{\theta}}^{\overline{\theta}} \bar{\pi}(\theta)\, dF(\theta) - \int_{\chi_2(t)}^{\theta_1} \left(\bar{\pi}(\theta) - \underline{\pi}(\theta)\right)\, dF(\theta) \\ &= c_2(\chi_2(t), \theta_1), \end{aligned} \tag{46}$$

where, $\chi_2(t)$ denotes the lowest remaining type in the inefficient sector (sector $B$) during the second regime. Using the same argument as in Section 4.1, the solution consists of the rate at which types change

$$\phi_2(\chi, \theta_1) = \frac{\bar{\pi}(\chi) - \underline{\pi}(\chi)}{\bar{\pi}'(\chi)} \frac{1}{\nu_2(\chi; \theta_1)} \tag{47}$$

where $\nu_2(\chi;\theta_1)$ is the price of a perpetuity in the current state and solves the ODE

$$\left(\rho + \gamma\, c_2(\chi,\theta_1)^{-1}\left(\bar{\pi}(\chi) - \underline{\pi}(\chi)\right) f(\chi)\,\phi_2(\chi,\theta_1)\right)\nu_2(\chi;\theta_1) \;= 1 + \phi_2(\chi,\theta_1)\,\nu_2'(\chi;\theta_1). \tag{48}$$

The boundary condition now becomes

$$\nu_2(\theta_1;\theta_1) = \rho^{-1}. \tag{49}$$

The value of an efficiently allocated unit of capital of quality $\theta$ in the second regime is therefore equal to

$$\bar{V}_2(\theta,\chi_2,\theta_1) = \nu_2(\chi_2,\theta_1)\,\bar{\pi}(\theta). \tag{50}$$

Next, we derive the value of an inefficiently allocated unit of capital during regime 2. It will be sufficient to compute this value evaluated at $\chi_2 = \underline{\theta}$ for all $\theta_1$, which is given by

$$\begin{aligned}
\xi_t\,\underline{V}_2(\theta,\underline{\theta},\theta_1) &= \int_t^{T_2(\theta,\theta_1)} \xi_s\,\underline{\pi}(\theta)\,ds + \int_{T_2(\theta,\theta_1)}^{\infty} \xi_s\,\bar{\pi}(\theta)\,ds \\
&= \int_t^{T_2(\theta,\theta_1)} \xi_s\,(\underline{\pi}(\theta) - \bar{\pi}(\theta))\,ds + \int_t^{\infty} \xi_s\,\bar{\pi}(\theta)\,ds \\
&= \xi_t \bar{V}_2(\theta,\underline{\theta},\theta_1) - \int_t^{T_2(\theta,\theta_1)} \xi_s\,(\bar{\pi}(\theta) - \underline{\pi}(\theta))\,ds
\end{aligned} \tag{51}$$

where $T_2(\theta,\theta_1) = \int_{\underline{\theta}}^{\theta} \frac{1}{\phi_2(y,\theta_1)}dy$ is the stopping rule used by a type $\theta$ seller in the second regime. Using a change a variables, equation (51) can be written as

$$\underline{V}_2(\theta,\underline{\theta},\theta_1) = \bar{V}_2(\theta,\underline{\theta},\theta_1) - (\bar{\pi}(\theta) - \underline{\pi}(\theta)) \int_{\underline{\theta}}^{\theta} \exp\left(-\rho\,T_2(y,\theta_1)\right)\left(\frac{c_2(y,\theta_1)}{c_2(\underline{\theta},\theta_1)}\right)^{-\gamma} \frac{1}{\phi_2(y,\theta_1)}\,dy \tag{52}$$

where instead of integrating over time, we integrate over types that switch before a type $\theta$ switches. Substituting in the expression for $T_2$, one can calculate $\underline{V}_2(\theta,\underline{\theta},\theta_1)$ in terms of $\bar{V}_2$ and $\phi_2$.

### A.2.2 First Regime

By the zero-profit condition, the price must equal the value of an efficiently allocated unit of capital in the first regime– denoted by $\bar{V}_1$ – satisfies

$$\xi_t\,\bar{V}_1(\theta,\chi_1(t)) = E\left[\int_t^{\tau} \xi_s\,\bar{\pi}(\theta)\,ds + \xi_\tau \underline{V}_2(\theta,\underline{\theta},\chi_1(\tau))\right].$$

Here, $\chi_1(t)$ denotes the lowest quality of capital remaining in the inefficient sector during the first regime (sector $A$). Because $\chi_1(t)$ must be monotonic in a fully-revealing equilibrium, we often omit $t$ arguments and write things in terms of the state variable $\chi_1$, using $\phi_1(\chi_1)$ to denote the rate of reallocation in the first regime. Aggregate consumption and output in the first regime is given by

$$c_1(\chi_1) \equiv \int_{\underline{\theta}}^{\chi_1} \bar{\pi}(\theta)\,dF(\theta) + \int_{\chi_1}^{\bar{\theta}} \underline{\pi}(\theta)\,dF(\theta).$$

From Lemma 2.1, the stochastic discount factor is $\xi_t = e^{-\rho t} c_1(\chi_1)^{-\gamma}$, which satisfies

$$\frac{d\xi_t}{\xi_t} = -\rho\, dt - \gamma\, c_1(\chi_1)^{-1} \frac{\partial}{\partial \chi} c_1(\chi_1)\phi_1(\chi_1) dt + \left( \left( \frac{c_2(\underline{\theta}, \chi_1)}{c_1(\chi_1)} \right)^{-\gamma} - 1 \right) dN_t,$$

$$E_t \left[ \frac{d\xi_t}{\xi_t} \right] = \underbrace{-\rho\, dt - \gamma\, c_1(\chi_1)^{-1} \frac{\partial}{\partial \chi} c_1(\chi_1)\phi_1(\chi_1) dt + \lambda \left( \left( \frac{c_2(\underline{\theta}, \chi_1)}{c_1(\chi_1(t))} \right)^{-\gamma} - 1 \right) dt}_{-r_1(\chi_1)\, dt}.$$

Define the discounted price process $\tilde{V}$, as

$$\begin{aligned} \tilde{V}(\theta, \chi_1) &\equiv c_1(\chi_1)^{-\gamma}\, \bar{V}_1(\theta, \chi_1) \\ &= E \left[ \int_t^\tau e^{-\rho(s-t)} c_1(\chi_1(s))^{-\gamma}\, \bar{\pi}(\theta)\, ds + e^{-\rho(\tau - t)} \left( c_2(\underline{\theta}, \chi_1(\tau)) \right)^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \chi_1(\tau)) \right]. \end{aligned}$$

Also, note that

$$\begin{aligned} \tilde{V}_\chi &\equiv \frac{d}{d\chi} \left( c_1(\chi)^{-\gamma}\, \bar{V}_1(\theta, \chi) \right) \\ &= c_1(\chi)^{-\gamma}\, \bar{V}_\chi(\theta, \chi) - \gamma\, c_1(\chi)^{-(1+\gamma)}\ \left( \bar{\pi}(\chi) - \underline{\pi}(\chi) \right) f(\chi)\, \bar{V}_1(\theta, \chi). \end{aligned}$$

By the martingale property, $\tilde{V}(\theta, \chi_1)$ satisfies the ODE

$$\rho\, \tilde{V} = c_1(\chi_1)^{-\gamma} \bar{\pi}(\theta) + \tilde{V}_\chi\, \phi_1(\chi_1) + \lambda \left( \left( c_2(\underline{\theta}, \chi_1) \right)^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \chi_1). - \tilde{V} \right)$$

Or, after substituting for $\tilde{V}$

$$r_1(\chi) \bar{V}_1(\theta, \chi) = \bar{\pi}(\theta) + \frac{\partial}{\partial \chi} \bar{V}_1(\theta, \chi)\, \phi(\chi) + \lambda \left( \frac{c_2(\underline{\theta}, \chi)}{c_1(\chi)} \right)^{-\gamma} \left( \underline{V}_2(\theta, \underline{\theta}, \chi) - \bar{V}_1(\theta, \chi) \right).$$

Since this is the value of an efficiently allocated unit of capital, the above equation holds only for $\theta \leq \chi$. For the boundary condition, consider what happens to an efficiently allocated unit of capital when all the capital has moved, but before the second shock hits, $\chi_1 = \overline{\theta}$ and $t < \tau$. The value of capital at the boundary must solve

$$\rho\, \bar{V}_1(\theta, \overline{\theta}) = \bar{\pi}(\theta) + \lambda \left( \left( \frac{c_2(\underline{\theta}, \overline{\theta})}{c_1(\overline{\theta})} \right)^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \overline{\theta}) - \bar{V}_1(\theta, \overline{\theta}) \right),$$

or, equivalently

$$\bar{V}_1(\theta, \overline{\theta}) = \frac{1}{\rho + \lambda} \bar{\pi}(\theta) + \frac{\lambda}{\rho + \lambda} \left( \frac{c_2(\underline{\theta}, \overline{\theta})}{c_1(\overline{\theta})} \right)^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \overline{\theta}).$$

Next, we solve for the value of an inefficient unit of capital. Following the same steps as before, the value of an inefficiently allocated unit $\underline{V}_1$ satisfies

$$r_1(\chi)\,\underline{V}_1(\theta,\chi) = \underline{\pi}(\theta) + \phi_1(\chi)\frac{\partial}{\partial\chi}\,\underline{V}_1(\theta,\chi)\, + \lambda\left(\frac{c_2(\underline{\theta},\chi)}{c_1(\chi)}\right)^{-\gamma}\left(\bar{V}_2(\theta,\underline{\theta},\chi) - \underline{V}_1(\theta,\chi)\right).$$

Zero profit requires that

$$P_1(\chi) = \bar{V}_1(\chi,\chi).$$

At the instant where type $\theta$ trades he has to be locally indifferent between waiting or not. So at the boundary, we have that

$$P_1(\chi) = \bar{V}_1(\chi,\chi) = \underline{V}_1(\chi,\chi)$$

and

$$\begin{aligned}\phi_1(\chi)\,\frac{\partial}{\partial\chi}\underline{V}_1(\theta,\chi)\bigg|_{\theta=\chi} &= \phi_1(\chi)\,\frac{d}{\partial\chi}P(\chi)\\ &= \left(\frac{\partial}{\partial\theta}\bar{V}_1(\theta,\chi)|_{\theta=\chi} + \frac{\partial}{\partial\chi}\bar{V}_1(\theta,\chi)|_{\theta=\chi}\right)\phi_1(\chi).\end{aligned}$$

Replacing the partials with respect to $\chi$ in the LHS and the RHS using the two ODEs for $\underline{V}_1$ and $\bar{V}_1$, we arrive at

$$\bar{\pi}(\chi) - \underline{\pi}(\chi) - \lambda\left(\frac{c_2(\underline{\theta},\chi)}{c_1(\chi)}\right)^{-\gamma}\left(\bar{V}_2(\theta,\underline{\theta},\chi) - \underline{V}_2(\theta,\underline{\theta},\chi)\right) = \left(\frac{\partial}{\partial\theta}\bar{V}_1(\theta,\chi)|_{\theta=\chi}\right)\phi_1(\chi).$$

Therefore, in equilibrium, the rate of reallocation is given by

$$\phi_1(\chi) = \max\left\{\frac{\bar{\pi}(\chi) - \underline{\pi}(\chi) - \lambda\left(\frac{c_2(\underline{\theta},\chi)}{c_1(\chi)}\right)^{-\gamma}\left(\bar{V}_2(\theta,\underline{\theta},\chi) - \underline{V}_2(\theta,\underline{\theta},\chi)\right)}{\frac{\partial}{\partial\theta}\bar{V}_1(\theta,\chi)|_{\theta=\chi}}, 0\right\}.$$

### A.2.3 Numerical Solution Method

The numerical solution also works by backward induction. Starting in period 2, we first solve for $\nu_2$ using equations (48) and (49). Using equation (47), we can then find the rate of reallocation in the second period. From this, we then solve for the equilibrium value functions in the second period ($\bar{V}_2$ and $\underline{V}_2$) using (50) and (52). After replacing equation (47) into (50) and (52), we obtain two non-linear ODEs in $\chi$, with an initial condition at $\chi = \underline{\theta}$. We solve these ODEs numerically using an explicit Runge-Kutta (4,5) formula – implemented in Matab's ode45 solver. Moving back to the first regime, we solve for the two value functions in the first period ($\bar{V}_1$ and $\underline{V}_1$) using the same methodology, while taking the solutions ($\bar{V}_2$ and $\underline{V}_2$) as given.

# B Data and Empirical Methodology

## B.1 Definitions and Data Description

Accounting data is from Compustat. Profitability (return to assets) is net income (Compustat: ni) divided by book assets (Compustat: at). Accounting variables in year $s$ refer to variables corresponding to fiscal year ending in calendar year $s$. Industry is 2-digit SIC code. Market capitalization at year $t$ is given by the absolute value of (CRSP: prc) times (CRSP: shrout) at the end of December of year $t$. Data on Stock returns is from CRSP. Stock return for year $t$ is the mean monthly return for calendar year $t$, annualized by multiplying it by 12. Data on IPOs and firm age is from Jay Ritter's website (http://bear.warrington.ufl.edu/ritter/ipodata.htm). We restrict the sample to those firms with non-missing observations on profitability, size, market capitalization, industry code, and book assets on the year of the IPO, leaving us with 6,004 firms (IPO events) covering the period 1975 to 2012. We winsorize al variables at the 0.5% at 99.5% percentiles using annual breakpoints.

## B.2 Results

The following table presents descriptive statistics for our variables of interest.

**Table 1:** Descriptive Statistics

| stats | Mean | SD | p10 | p25 | p50 | p75 | p90 |
|---|---|---|---|---|---|---|---|
| Firm age at IPO | 13.51 | 18.19 | 2.00 | 3.00 | 7.00 | 15.00 | 32.00 |
| Book Assets, log | 3.74 | 1.70 | 1.58 | 2.62 | 3.72 | 4.76 | 5.91 |
| Profitability (ROA) | -0.06 | 0.31 | -0.37 | -0.10 | 0.03 | 0.09 | 0.15 |
| Market Capitalization, log | 11.32 | 1.53 | 9.33 | 10.21 | 11.31 | 12.33 | 13.32 |
| Returns | 0.10 | 1.38 | -1.34 | -0.61 | 0.01 | 0.67 | 1.56 |

**Table 2:** Firm Age at IPO versus Ex-post Profitability and Stock Returns

| Horizon | A. Profitability (ROA) | | | B. Stock Returns | | |
|---|---|---|---|---|---|---|
| (year after IPO) | (I) | (II) | (III) | (I) | (II) | (III) |
| 1 | 0.102 | 0.030 | 0.032 | 0.040 | 0.018 | 0.021 |
| | [10.54] | [2.82] | [2.25] | [2.04] | [1.06] | [1.32] |
| 2 | 0.124 | 0.055 | 0.057 | 0.002 | 0.005 | 0.005 |
| | [9.56] | [4.68] | [4.58] | [0.14] | [0.34] | [0.40] |
| 3 | 0.127 | 0.054 | 0.051 | 0.005 | 0.011 | 0.005 |
| | [8.76] | [4.62] | [3.37] | [0.23] | [0.73] | [0.23] |
| 4 | 0.114 | 0.043 | 0.055 | -0.013 | 0.001 | 0.006 |
| | [8.18] | [3.44] | [3.22] | [-0.99] | [0.07] | [0.38] |
| 5 | 0.148 | 0.035 | 0.071 | 0.002 | 0.016 | 0.008 |
| | [4.64] | [1.53] | [1.81] | [0.10] | [0.98] | [0.39] |
| Controls | | | | | | |
| Book Assets, log | | y | y | | y | y |
| Market Capitalization, log | | y | y | | y | y |
| ROA | | y | y | | y | y |
| IPO year | y | y | | y | y | |
| INDxIPO year | | y | y | | y | y |

Panels A and B of the Table present estimates of the coefficient $b$ from the following empirical specifications

$$ROA_{ft+k} = b\,\log(1 + A_{ft}) + c\,Z_{ft} + u_{ft+k}$$

and

$$R_{ft+k} = b\,\log(1 + A_{ft}) + c\,Z_{ft} + u_{ft+k},$$

respectively. Columns (I) to (III) present results with different controls. Here, $A_{ft}$ is age of the firm at the time of the IPO, and $ROA_{fs}$ and $R_{fs}$ is profitability and stock returns, respectively, for firm $f$ in year $s$. We examine horizons of up to five years following the IPO, $s = t \ldots t + k$. The IPO year corresponds to year $t$. We include a vector of controls $Z$ that, depending on the specification, includes IPO-year fixed effects, firm profitability at year $t$, firm market capitalization at the year $t$, book assets at year $t$, and industry-specific IPO year dummies. We include $t$-statistics in brackets computed using standard errors clustered by IPO year.